

Isospin effects and the density dependence of the nuclear symmetry energy

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The density dependence of the nuclear symmetry energy is inspected using the Statistical Multifragmentation Model with Skyrme effective interactions. The model consistently considers the expansion of the fragments' volumes at finite temperature at the freeze-out stage. By selecting parameterizations of the Skyrme force that lead to very different equations of state for the symmetry energy, we investigate the sensitivity of different observables to the properties of the effective forces. Our results suggest that, in spite of being sensitive to the thermal dilation of the fragments' volumes, it is difficult to distinguish among the Skyrme forces from the isoscaling analysis. On the other hand, the isotopic distribution of the emitted fragments turns out to be very sensitive to the force employed in the calculation.

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Investigations on the density dependence of the symmetry energy in the nuclear multifragmentation process have been stimulated by the discovery [1, 2] that the ratio R_{21} between the experimental yields $Y(A, Z)$ of a fragment of mass and atomic numbers A and Z , respectively, produced in similar reactions with different isospin compositions, henceforth labeled '1' and '2', follows a scaling law [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. This isoscaling law provides parameters α and β that are determined from the property

$$R_{21} \equiv \frac{Y_2(A, Z)}{Y_1(A, Z)} = C \exp(\alpha N + \beta Z) \quad (1)$$

where C is a normalization constant [1].

Studies of the density dependence of the symmetry energy have used Eq. (1) to: (i) probe the dependence of statistical models on the symmetry energy [7, 8, 9, 13] and (ii) to probe the isospin composition of the region emitting the fragments [3, 17, 18]. In this paper, we are concerned with the former issue. The motivation for statistical model studies arises from the relationship between the isoscaling parameter α and the nuclear symmetry energy, whose leading term at low temperatures was shown to be [15, 16]

$$\alpha \approx 4C_{\text{sym}} [(Z_1/A_1)^2 - (Z_2/A_2)^2] / T, \quad (2)$$

where C_{sym} denotes the symmetry energy coefficient [6], T is the temperature of the system, Z_i and A_i , $i = 1, 2$,

correspond, respectively, to the atomic and mass numbers of the decaying source.

To be useful, reactions '1' and '2' should be chosen to produce systems at approximately the same temperature and density. The ratio in Eq. (1) involves yields of the same isotope; distorting effects associated with the deexcitation of the primordial hot fragments may be similar in the two reactions and lead to an approximate cancellation in the ratio. Theoretical calculations support this assumption for primary distributions calculated from equilibrium statistical models [13, 14, 15]. Therefore, measurements of the isoscaling parameters, through fits based on Eq. (1), may provide valuable information on the symmetry energy.

This assumption has extensively been exploited in many works [7, 8, 9, 10, 11, 12, 13] which employed the Statistical Multifragmentation Model (SMM) [19]. In order to reproduce the measured α parameter, C_{sym} has been appreciably reduced, compared with the values usually adopted in this model. The main conclusion of those works [7, 8, 9, 10, 11, 12, 13] is that there seems to be an important decrease of the symmetry energy at low densities.

Although this result is reasonable on physical grounds, other studies [4, 5, 6] also provided a sound explanation to this apparent reduction of C_{sym} . They suggest that surface effects associated with the symmetry energy, not considered in Refs. [7, 8, 9, 10, 11, 12, 13], may also lead to significant reduction of the α parameter. This explanation seems to be more reasonable since the model

used in all these studies [4, 5, 6, 7, 8, 9, 10, 11, 12, 13] is based on binding energy formulae that evaluate the isoscalar volume, surface and Coulomb terms at the ground state (saturation) density. Although the volume occupied by the total system is much larger than that of the ground state source V_0 , the lower density values used in these statistical calculations are due to the space between the fragments. Therefore, in this scenario, there should be no sensitivity to the density dependence of the bulk symmetry energy.

In this work we investigate this issue in a consistent approach in which the properties of the fragments are calculated through the Thomas-Fermi approximation (TFA) at finite temperature. This version of the SMM, named SMM-TF, was presented in Ref. [20]. In this way, the changes to the fragments' energies and densities at the freeze-out stage are consistently obtained in the framework of the TFA. Thus, the total Helmholtz free energy F of a given partition mode is written in the same form as in the Improved Statistical Multifragmentation Model (ISMM) [21], also used in this work:

$$F(T) = \frac{C_{\text{Coul}}}{(1+\chi)^{1/3}} \frac{Z_0^2}{A_0^{1/3}} + F_{\text{trans}}(T) \quad (3)$$

$$+ \sum_{A,Z} N_{A,Z} \left[-B_{A,Z} + f_{A,Z}^*(T) - \frac{C_{\text{Coul}}}{(1+\chi)^{1/3}} \frac{Z^2}{A^{1/3}} \right]$$

where

$$F_{\text{trans}} = -T(M-1) \log(V_f/\lambda_T^3) + T \log(g_0 A_0^{3/2}) \quad (4)$$

$$- T \sum_{A,Z} N_{A,Z} \left[\log(g_{A,Z} A^{3/2}) - \frac{1}{N_{A,Z}} \log(N_{A,Z}!) \right].$$

In the above Eqs., C_{Coul} denotes the Coulomb coefficient of the mass formula [21], A_0 and Z_0 are the mass and atomic numbers of the decaying source, $B_{A,Z}$ represents the binding energy of the fragment, $N_{A,Z}$ stands for its multiplicity and $M = \sum_{A,Z} N_{A,Z}$. The freeze-out volume $V_\chi = V_0(1+\chi)$ is kept fixed ($\chi = 2$) for all the values of the source's excitation energy E^* . The factor $M-1$, rather than M , as well as $T \log(g_0 A_0^{3/2})$, in the translational contribution to the free energy, arise from the subtraction of the center of mass motion from the partition function of the total system. The spin degeneracy factor is denoted by $g_{A,Z}$, $\lambda_T = \sqrt{2\pi\hbar^2/m_n T}$ corresponds to the thermal wave-length, and m_n is the nucleon mass. As in the ISMM, the free volume reads

$$V_f(T) = V_0(1+\chi) - \sum_{A,Z} N_{A,Z} V_{A,Z}(T), \quad (5)$$

where $V_{A,Z}(T)$ is the volume occupied by each fragment. However, in the SMM-TF, $V_{A,Z}(T)$ is given by the TFA

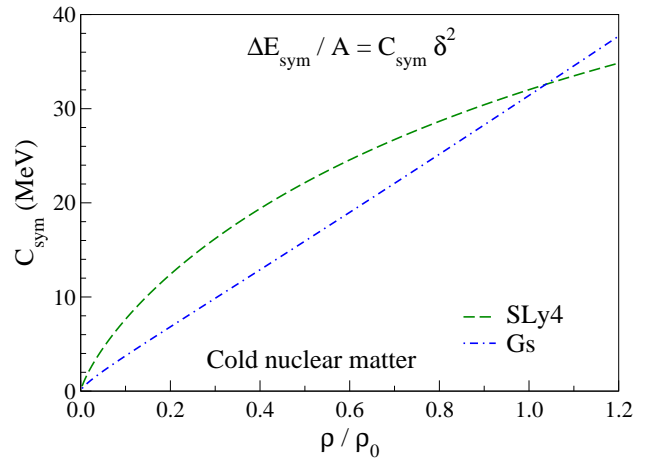


FIG. 1: (Color online) Symmetry energy coefficient of cold nuclear matter for the SLy4 and Gs Skyrme forces as a function of the density ρ . For details, see the text.

[20] and therefore it differs from the ground state value used in the ISMM. The internal Helmholtz free energy of the fragment $f_{A,Z}^*$ in the SMM-TF model is also given by this microscopic approach. These are the only two differences between the SMM-TF and the ISMM, as discussed in Ref. [20]. All the other ingredients are the same.

Owing to the plethora of Skyrme forces in the literature [22, 23], we do not perform a detailed study using all the existing parameterizations. Instead, we selected two of them which give distinct Equations of State (EOS's) for the symmetry energy, *i. e.*, the Gs [24] and the SLy4 [25] forces. Figure 1 shows the density dependence of the symmetry energy coefficient of cold nuclear matter, $\delta E_{\text{sym}}/A = C_{\text{sym}}\delta^2$, where $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$ and ρ_n (ρ_p) is the neutron (proton) density. One sees that, although both forces agree for densities close to the saturation value (ρ_0), the differences at lower densities can be appreciably large. The Gs and SLy4 forces provide examples of a strongly density dependent (stiff) and a weakly density dependent (soft) symmetry energy, respectively. Both have bulk isoscalar incompressibility moduli in the range of 230 to 250 MeV [26]. Therefore, they are well suited to the present study.

We confine our attention to the ^{40}Ar , $^{40}\text{Ca} + ^{58}\text{Ni}$ systems, for which measurements of the isoscaling parameter α have been reported recently [13]. The decaying sources considered in the calculations below correspond to 80% of the compound systems, as 20% of the matter is removed in order to take the preequilibrium emission into account. To make a connection to the work of Ref. [13], we adopt in the following their assumption that the Z/A of the source is the same as the original system.

We begin our discussion by examining whether the isoscaling property should still hold if the thermal expansion of the fragments' volumes is taken into account. The microcanonical treatment employed in the SMM-TF

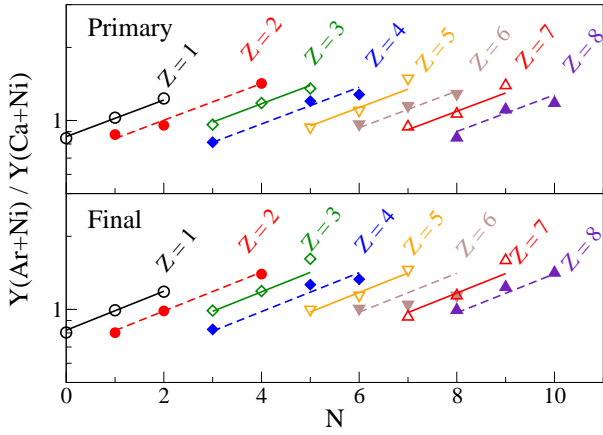


FIG. 2: (Color online) Isoscaling predicted by the SMM-TF using the SLy4 for $E^*/A = 6.0$ MeV. In the upper panel the yields of the hot primary fragments are used whereas the lower panel displays the results after the deexcitation of these fragments. The lines correspond to the best fit of the results using Eq. (1). For details, see the text.

does not allow one to derive analytical expressions to investigate this issue. Furthermore, the dependence of the free volume on the species multiplicities, Eq. (5), lead to highly non-linear terms in the Helmholtz free energy, rendering the traditional grand-canonical formulas [6] invalid. By minimizing F with respect to the multiplicities, as is done in Ref. [6], one may nevertheless obtain formal expressions which suggest that the isoscaling property should still be observed in this case.

This is indeed found in our numerical microcanonical SMM-TF calculations, as is illustrated in Fig. 2, which shows R_{21} for the SLy4 force and $E^*/A = 6.0$ MeV. Similar results are obtained for the Gs force and for other excitation energies. The magnitude of the corrections due to the deexcitation of the hot primary fragments can be estimated by comparing the upper and lower panels of Fig. 2. Here, we simulated the decay with a simplified Monte Carlo Weisskopf model, which includes the emission of nuclei up to oxygen. The parameterization of the cross-section for the inverse reaction was taken from Ref. [27]. Following Ref. [28], we calculate the density of states from the entropies associated with $f_{A,Z}^*$. This provides a consistent link between the primary stage and the deexcitation process. In agreement with previous calculations [13, 15], our results also suggest that α is not strongly sensitive to the deexcitation of the primary fragments.

Figure 3 shows the comparison between the α values obtained in the different SMM models used in this work and the available experimental data [13]. The results corresponding to the primary fragments (top panel of this picture) reveal that α is fairly sensitive to the thermal dilatation of the fragments' volumes as the behavior of α in the SMM-TF calculations is different from that given by the ISMM. Switching from the SLy4 to the Gs

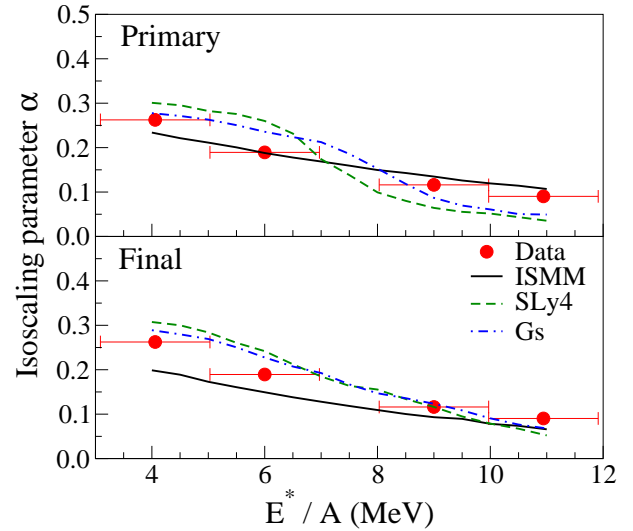


FIG. 3: (Color online) Isoscaling parameter α as a function of the excitation energy of the source, for different Skyrme forces and the ISMM, before and after the deexcitation of the primary fragments. The data are taken from Ref. [13]. For details, see the text.

force leads to small differences in α , primarily within the small energy range $7.0 \text{ MeV} < E^*/A < 8.5 \text{ MeV}$. This corresponds to the region where the average fragments' density is approximately $0.65\rho_0$. For the ISMM, investigations of the connection between α and C_{sym} at finite temperatures show that Eq. (2) can be fairly inaccurate at high temperatures, although the main conclusion that both quantities are strongly correlated remains valid [29].

It is interesting to examine whether comparisons of the SMM-TF calculations to experimental fragmentation data can clarify questions regarding the adequacy of the model and the effective forces selected to describe the multifragment emission. The ISMM results seem to follow the experimental trends more closely than the SMM-TF calculations. This picture does not survive the deexcitation of the primary fragments. Although the changes to α are small, the values predicted by the ISMM are systematically lowered, so that after secondary decay it lies below the data and the SMM-TF. The differences between the calculations with the SLy4 and the Gs forces are very small after the decay of the primary fragments and our results suggest that it is difficult to distinguish between them from the isoscaling analysis.

Distinct Skyrme forces lead to different values of $V_{A,Z}(T)$, according to their properties. This directly affects the Helmholtz free energy of the system through changes in V_f and indirectly through changes in the internal free energies of the fragments. Therefore, it suggests that the fragment composition should be very sensitive to the thermal expansion of $V_{A,Z}$. This is indeed observed in Fig. 4, which displays the isotopic distribution of selected primary hot nuclei for the breakup of the Ar + Ni sys-

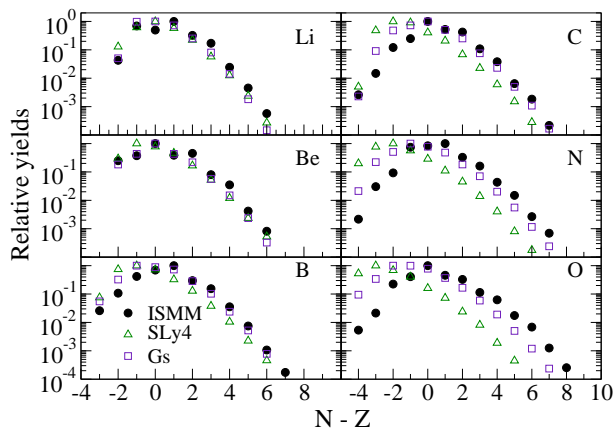


FIG. 4: (Color online) Isotopic distribution of selected primary fragments for the Ar + Ni system at $E^*/A = 8.5$ MeV. For details see the text.

tem at $E^*/A = 8.5$ MeV. It has been shown in Ref. [20] that, at low energies, the isotopic distributions predicted by the SMM-TF are narrower than those given by the ISMM. However, they are very different at higher excitation energies, as shown in Fig. 4, where, for this system, the peak of the distribution shifts toward the proton rich isotopes. The effect is enhanced for higher excitation energies and it is more pronounced in the case of the SLy4 force than for the Gs force. We have checked that, although these trends remain true, the differences between the Gs and the SLy4 forces are reduced after the decay of the primary fragments whereas they remain large enough to clearly distinguish between the ISMM and the SMM-TF calculations. However, in order to draw precise conclusions, a deexcitation treatment that explicitly takes the feeding between known discrete states of these fragments into account, such as that presented in Ref. [21], should be developed and applied to the SMM-TF.

In summary, we have tested, self-consistently, the sensitivity of equilibrium multifragmentation theories to the density dependence of the symmetry energy. Our statistical calculations, which consistently incorporate Skyrme effective interactions, suggest that the corresponding predicted differences in the isoscaling parameter α are not large enough to allow one to distinguish between symmetry energies with very different density dependencies. This observation does not impact the use of α to probe the asymmetry of the emitting system, a use that has been instrumental for investigations of the density dependent symmetry energy [3, 17, 18]. On the other hand, the isotopic distribution of the fragments produced in the reactions retains some sensitivity to the effective force employed in the calculations. We believe that constraining the key SMM-TF inputs, *i.e.* the source composition and the excitation energy, through experimental values of additional observables besides the isoscaling parameters, will be critical to precision comparisons aimed at

constraining the symmetry energy with isotopic distributions.

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- [1] H. S. Xu, M. B. Tsang, T. X. Liu, X. D. Liu, W. G. Lynch, W. P. Tan, A. Vander Molen, G. V. A. Wagner, H. F. Xi, C. K. Gelbke, et al., Phys. Rev. Lett. **85**, 716 (2000).
 - [2] M. B. Tsang, W. A. Friedman, C. K. Gelbke, W. G. Lynch, G. Verde, and H. S. Xu, Phys. Rev. Lett. **86**, 5023 (2001).
 - [3] A. Ono, P. Danielewicz, W. A. Friedman, W. G. Lynch, and M. B. Tsang, Phys. Rev. C **68**, 051601(R) (2003).
 - [4] Ad. R. Raduta and F. Gulminelli, Phys. Rev. C **75**, 024605 (2007).
 - [5] Ad. R. Raduta and F. Gulminelli, Phys. Rev. C **75**, 044605 (2007).
 - [6] S. R. Souza, M. B. Tsang, R. Donangelo, W. G. Lynch, and A. W. Steiner, Phys. Rev. C **78**, 014605 (2008).
 - [7] A. Le Fèvre, G. Auger, M. L. Begemann-Blaich, N. Bellaïze, R. Bittiger, F. Bocage, B. Borderie, R. Bougault, B. Bouriquet, J. L. Charvet, et al., Phys. Rev. Lett. **94**, 162701 (2005).
 - [8] D. V. Shetty, S. J. Yennello, and G. A. Souliotis, Phys. Rev. C **76**, 024606 (2007).
 - [9] A. S. Botvina, N. Buyukcizmeci, M. Erdogan, J. Lukasik, I. N. Mishustin, R. Ogul, and W. Trautmann, Phys. Rev. C **74**, 044609 (2006).
 - [10] S. Kowalski, J. B. Natowitz, S. Shlomo, R. Wada, K. Hagel, J. Wang, T. Materna, Z. Chen, Y. G. M. L. Qin, A. S. Botvina, et al., Phys. Rev. C **75**, 014601 (2007).
 - [11] G. A. Souliotis, A. S. Botvina, D. V. Shetty, A. L. Keksis, M. Jandel, M. Veselsky, and S. J. Yennello, Phys. Rev. C **75**, 011601(R) (2007).
 - [12] N. Buyukcizmeci, R. Ogul, and A. S. Botvina, Eur. Phys. J. A **25**, 57 (2005).
 - [13] J. Iglío, D. V. Shetty, S. J. Yennello, G. A. Souliotis, M. Jandel, A. L. Keksis, S. N. Soisson, B. C. Stein, S. Wuenschel, and A. S. Botvina, Phys. Rev. C **74**, 024605 (2006).
 - [14] S. R. Souza, R. Donangelo, W. G. Lynch, W. P. Tan, and M. B. Tsang, Phys. Rev. C **69**, 031607(R) (2004).
 - [15] M. B. Tsang, C. K. Gelbke, X. D. Liu, W. G. Lynch, W. P. Tan, G. Verde, H. S. Xu, W. A. Friedman, R. Donangelo, S. R. Souza, et al., Phys. Rev. C **64**, 054615 (2001).
 - [16] A. S. Botvina, O. V. Lozhkin, and W. Trautmann, Phys. Rev. C **65**, 044610 (2002).
 - [17] M. B. Tsang, T. X. Liu, L. Shi, P. Danielewicz, C. K. Gelbke, X. D. Liu, W. G. Lynch, W. P. Tan, G. Verde, A. Wagner, et al., Phys. Rev. Lett. **92**, 062701 (2004).
 - [18] W. P. Tan, B.-A. Li, R. Donangelo, C. K. Gelbke, M.-J. van Goethem, X. D. Liu, W. G. Lynch, S. Souza, M. B.

- Tsang, G. Verde, et al., Phys. Rev. C **64**, 051901(R) (2001).
- [19] J. P. Bondorf, A. S. Botvina, A. S. Iljinov, I. N. Mishustin, and K. Sneppen, Phys. Rep. **257**, 133 (1995).
- [20] S. R. Souza, B. V. Carlson, R. Donangelo, W. G. Lynch, A. W. Steiner, and M. B. Tsang, Phys. Rev. C **79**, 054602 (2009).
- [21] W. P. Tan, S. R. Souza, R. J. Charity, R. Donangelo, W. G. Lynch, and M. B. Tsang, Phys. Rev. C **68**, 034609 (2003).
- [22] J. Rikowska Stone, J. C. Miller, R. Koncewicz, P. D. Stevenson, and M. R. Strayer, Phys. Rev. C **68**, 034324 (2003).
- [23] P. Danielewicz and J. Lee, Nucl. Phys. **A818**, 36 (2009).
- [24] J. Friedrich and P.-G. Reinhard, Phys. Rev. C **33**, 335 (1986).
- [25] F. Douchin and P. Haensel, Phys. Lett. **B485**, 107 (2000).
- [26] G. Colò, N. Van Giai, J. Meyer, K. Bennaceur, and P. Bonche, Phys. Rev. C **70**, 024307 (2004).
- [27] A. S. Iljinov, I. N. Mishustin, J. P. Bondorf, R. Donangelo, and K. Sneppen, Nucl. Phys. **A475**, 663 (1987).
- [28] H. W. Barz, J. P. Bondorf, R. Donangelo, I. N. Mishustin, and H. Schulz, Nucl. Phys. **A448**, 753 (2008).
- [29] S. R. Souza, M. Tsang, B. V. Carlson, R. Donangelo, W. G. Lynch, and A. W. Steiner (2009), nucl-th/0906.4375.