Estimating the X-Ray Dose Rate from the MARATHON Tritium Target

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1 Introduction

1.1 Motivation

The JLab MARATHON collaboration has been approved to perform an experiment using an unpolarized 11 GeV electron beam scattered from unpolarized tritium (^{3}H) & Helium-3 (^{3}He) targets. The scattered electrons will be detected within a kinematic regime where nucleons are best described as a collection of quasi-free quarks (i.e. deep inelastic scattering). The broad goals of this experiment are to:

1. study the nuclear medium modifications to the proton & neutron unpolarized structure function F_2 (i.e. the "EMC" effect) within ^{3}H & ^{3}He nuclei

2. extract the ratio of the down to up quark momentum distributions within nucleons in a kinematic regime dominated by valence quarks (i.e. at intermediate to high x).

One notable strength of this experiment is that only cross section ratios will be measured, thereby significantly reducing sensitivity to several sources of systematic uncertainties.

In order to perform this experiment, it is necessary to construct a high-pressure gaseous tritium target. This introduces unique radiological hazards that must be understood & mitigated before the target is built & installed. One of these hazards is the production of relatively soft x-rays from the beta decay of the tritium nuclei ($t_{1/2} = 12.32$ y and $Q = 18.6$ keV). X-rays are produced when the beta decay electron undergoes bremsstrahlung either as a part of the beta decay process (i.e. "internal" bremsstrahlung - InB) or after wards while traversing the surrounding material (i.e. "external" bremsstrahlung) - ExB. The goal of this document is to estimate the x-ray dose rate for the tritium target proposed by the MARATHON collaboration. The guiding philosophy of the following calculations

is to make only those approximations that *overestimate* the dose rate. Finally, throughout this document, we'll set the speed of light to unity $c = 1$.

1.2 Overview

The target is a 25 cm long cylindrical aluminum vessel with a 1.25 cm inner diameter and 460 micron thickness filled with $P = 10$ atm of molecular tritium at room temperature ($T = 298$ K). This corresponds to an outer surface area of 100 cm^2, inner volume of $V = 31$ cm^3 and equivalent molecular hydrogen mass density of 0.82 mg/cm^3. Given a half life of $t_{1/2} = 12.32$ yr $= 3.888 \times 10^8$ s, the activity of tritium can be calculated as:

$$\frac{N_p}{\tau} = \left[\frac{\log(2)}{t_{1/2}}\right]\left[\frac{2N_A}{3.7\times10^{10}\ \text{decays}/(\text{s}\cdot\text{Ci})}\right]\left[\frac{PV}{RT}\right] = 740\ \text{Ci} \tag{1}$$

where $N_A = 6.022 \times 10^{23}$/mol is the Avogadro constant and $R = 8.3145$ J/mol/K is the universal gas constant. The production and stopping of electrons due to tritium inside this target is a complicated multistage process. In this section, we'll enumerate the assumptions and approximations made in this calculation.

First, electrons and InB photons are produced via beta decay. In principle, both of these particles can lose energy traversing the tritium in the target. The electron stopping power of tritium for 10 keV electrons is 51 keV $\cdot$ cm^2/mg [1]. Inside the target, this corresponds to 42 keV/cm. Naively this would indicate that the electrons are completely stopped before reaching the wall of the vessel. However, almost all of this stopping power is due to collisions with atomic electrons in the tritium gas. Even if all of the original beta decay electrons are fully stopped, the secondary electrons ejected by the beta decay electrons carry away the energy released via the original beta decay. Therefore, we conclude that the "stopping" simply results in a reshaping of the beta spectrum without reducing its integrated intensity. Since this is difficult to calculate, we'll simply use the original beta spectrum as the distribution of electron energies that is incident on the Al

wall of the vessel. Furthermore, since the Z-density-thickness product for the Al ($Z = 13$ & $\rho = 2.7$ g/cm^3) wall is 2000 larger than for the tritium, we'll ignore the energy loss & production of x-rays due bremsstrahlung from the tritium gas itself. The photon absorption cross section of 10 keV photons in tritium is 0.39 cm^2/g. Since this corresponds to an attention length of 32 m, we'll ignore the attenuation of InB photons due to the tritium gas before reaching the Al wall. In summary, both the electron and InB are effectively unattenuated by the tritium gas before reaching the Al wall.

Next, the electrons and InB photons traverse the Al wall. We'll assume that the electrons are completely stopped by the Al wall and produce an external bremsstrahlung spectrum given by the Kramers' formula. The photon absorption cross section of 10 keV photons in Al is 26 cm^2/g. Since this corresponds to an attention length of 140 microns for solid Al, we'll include the attenuation of InB photons due to the Al wall. Finally, we'll account for the attenuation of different "thicknesses" of air when estimating the dose rate at different distances from the target.

2 Beta Decay Spectrum

The number of β particles released during beta decay per unit time per unit β energy is given by:

$$\frac{d^2N_\beta}{dtdW} = \left(\frac{N_p}{\tau}\right)\frac{F(Z_d, W)W(W_0 - W)^2 p}{mQ^4 I_0} \tag{2}$$

where N_β & N_p are the number of beta particles & parent nuclei, t is time, W is the β energy, p is the β momentum, m is the rest mass of the β (electron), W_0 is the maximum possible energy of the β particle, $\tau = t_{1/2}/\log(2)$ is the lifetime of the parent nucleus, $F(Z, W)$ is the (unitless) Fermi function, Z_d is the atomic number of the daughter nucleus, and I_0 is a unitless normalization parameter given by:

$$I_0 = \frac{1}{mQ^4}\int_m^{W_0} F(Z_d, W)W(W_0 - W)^2 p\, dW \tag{3}$$

In order to explicitly evaluate the normalization parameter I_0, we'll first note the following kinematic relationships:

$$W = \sqrt{m^2 + p^2} \qquad v = \frac{p}{W} \qquad E = W - m \qquad E_0 = Q \rightarrow W_0 = Q + m \tag{4}$$

where v, E, & E_0 are the velocity, kinetic energy, & maximum kinetic energy of the β particle and Q is the total energy released during the beta decay. For our purposes, we will use the following common approximation ($\alpha Z \ll 1$) for the Fermi function:

$$F(Z, W) = \frac{2\pi\eta}{1 - \exp(-2\pi\eta)} \quad \& \quad \eta = \frac{\alpha Z}{v} \quad \rightarrow \quad F(Z_d, W) \approx 1 + \frac{\pi\alpha Z_d W}{p} \tag{5}$$

Note that this form of the Fermi function is just the nonrelativistic form of the Coulomb correction, which is also known as the Sommerfeld-Elwert factor. This integral can be written in closed form as:

$$I_0 = \frac{1}{30q^4}\left[s^3(x^2+4) - \left(\frac{15x}{2}\right)\left[sx - \log(x+s)\right] + \pi\alpha Z_d(x-1)^3(x^2+3x+6)\right] \tag{6}$$

where we've introduced two new unitless parameters:

$$q = \frac{Q}{m} \qquad x = 1 + q \qquad s = \sqrt{x^2 - 1} = \sqrt{2q + q^2} \tag{7}$$

Using $m = 511$ keV and $\alpha = 1/137$, we find that, for ^{3}H beta decay, $Q = 18.6$ keV, $Z_d = 2$, $x = 1 + 3.64 \times 10^{-2}$, $s = 0.272$, and $I_0 = 1.57$.

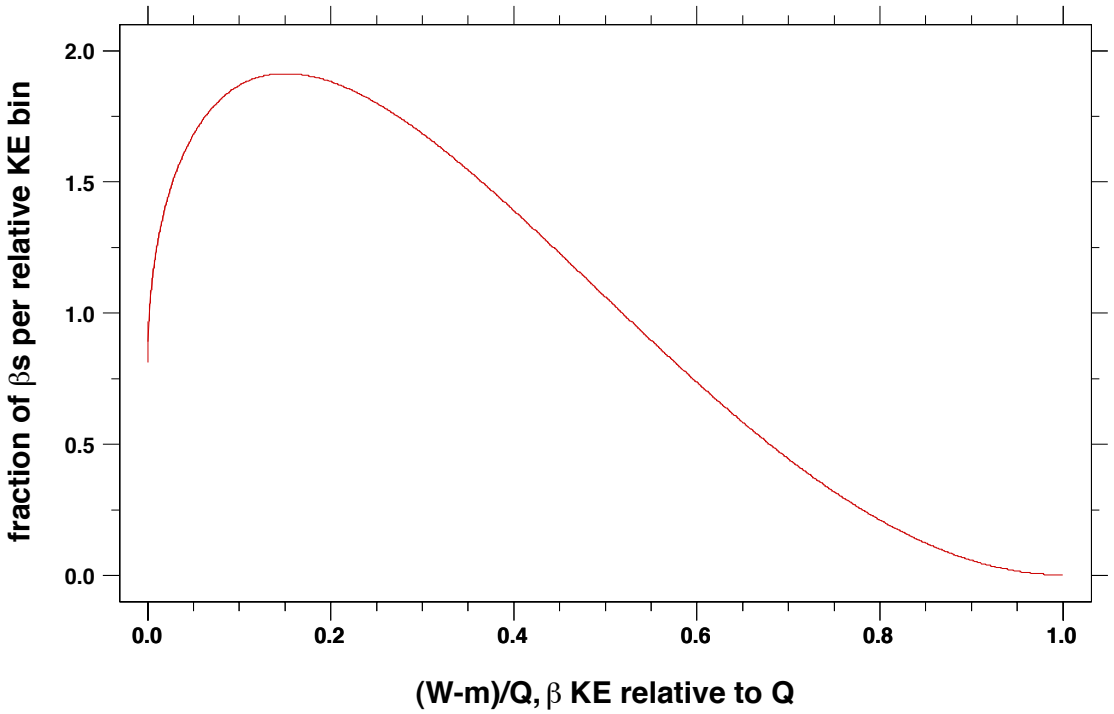

Figure 1: Beta Spectrum

3 Bremsstrahlung Spectrum

3.1 General

In general, the bremsstrahlung spectrum is simply the convolution of the beta spectrum with the bremsstrahlung spectral function:

$$\frac{d^2N_\gamma}{dtdk} = \int_{W_1}^{W_2} \left[\frac{d^2N_\beta}{dtdW}\right] \times \left[\frac{d^2N_\gamma}{dN_\beta dk}\right] dW = \left(\frac{N_p}{\tau}\right)\left(\frac{q}{k}\right)\left(\frac{\alpha}{\pi I_0}\right) S(k) \tag{8}$$

where k is the photon energy relative to the electron mass, W_1 & W_2 are the minimum & maximum beta energies, N_γ is the number of photons, the bremsstrahlung spectral function is the number of photons produced per beta particle per unit photon energy, and $S(k)$ is the unitless bremsstrahlung spectral function.

3.2 Internal Bremsstrahlung

Lewis & Ford [2] (NB: the sign error for the I_2 integral in p. 760 on the $W_0^2 x$ and W_0^2 terms on the last line - the signs should be reversed i.e. $+7x/6 - 7/9 \rightarrow -7x/6 + 7/9$) calculated the unitless internal bremsstrahlung spectral function as:

$$\frac{Q^5}{m^5} S(k) = \frac{1}{m^5} \int_m^{W_0 - k} (W_0 - k - W)^2 \left[1 + \frac{\pi \alpha Z_d W}{p}\right] \left[(W^2 + W_e^2) \log(W + p) - 2pW_e\right] \, dW \tag{9}$$

where $W_e = W + km$. This integral can be written in closed form as:

$$\begin{aligned}
S(k) &= \frac{I_1 + \pi \alpha Z_d I_2}{q^5} && (10)\\
I_1 &= + \left[\frac{x^2 \bar{x}}{3}\left(\bar{x}^2 + \frac{3}{2}\right) - \frac{x}{2}\left(\bar{x}^4 + \bar{x}^2 - \frac{1}{8}\right) + \frac{\bar{x}}{30}\left(7\bar{x}^4 - \frac{45}{8}\right)\right] \log(\bar{x} + \bar{s}) \\
&\quad - \left[\frac{x^2}{18}\left(11\bar{x}^2 + 4\right) - \frac{x\bar{x}}{8}\left(7\bar{x}^2 + \frac{1}{2}\right) + \frac{1}{1800}\left(689\bar{x}^4 - \frac{1021\bar{x}^2}{2} - 96\right)\right] \bar{s} && (11)\\
I_2 &= -\frac{x^2}{18}\left(11\bar{x}^3 - 18\bar{x}^2 + 21\bar{x} - 14\right) \\
&\quad + \frac{x}{8}\left(7\bar{x}^4 - 16\bar{x}^3 + 49\bar{x}^2 - 48\bar{x} + 8\right) \\
&\quad - \frac{689\bar{x}^5}{1800} + \bar{x}^4 - \frac{2059\bar{x}^3}{360} + \frac{67\bar{x}^2}{9} - \frac{61\bar{x}}{15} + \frac{388}{225} \\
&\quad - \frac{1}{2}\left[x^2\bar{x} - 3x\left(\bar{x}^2 + \frac{1}{4}\right) + 2\bar{x}\left(\bar{x}^2 + \frac{9}{8}\right)\right] \left[\log(\bar{x} + \bar{s})\right]^2 \\
&\quad + \frac{1}{6}\left[2x^2(\bar{x}^2 + 2) - 3x\bar{x}\left(\bar{x}^2 + \frac{13}{2}\right) + \frac{1}{5}\left(7\bar{x}^4 + \frac{177\bar{x}^2}{2} + 32\right)\right] \bar{s} \log(\bar{x} + \bar{s}) && (12)
\end{aligned}$$

where we've introduced two new unitless parameters:

$$\bar{x} = x - k = 1 + q - k \qquad \bar{s} = \sqrt{\bar{x}^2 - 1} \tag{13}$$

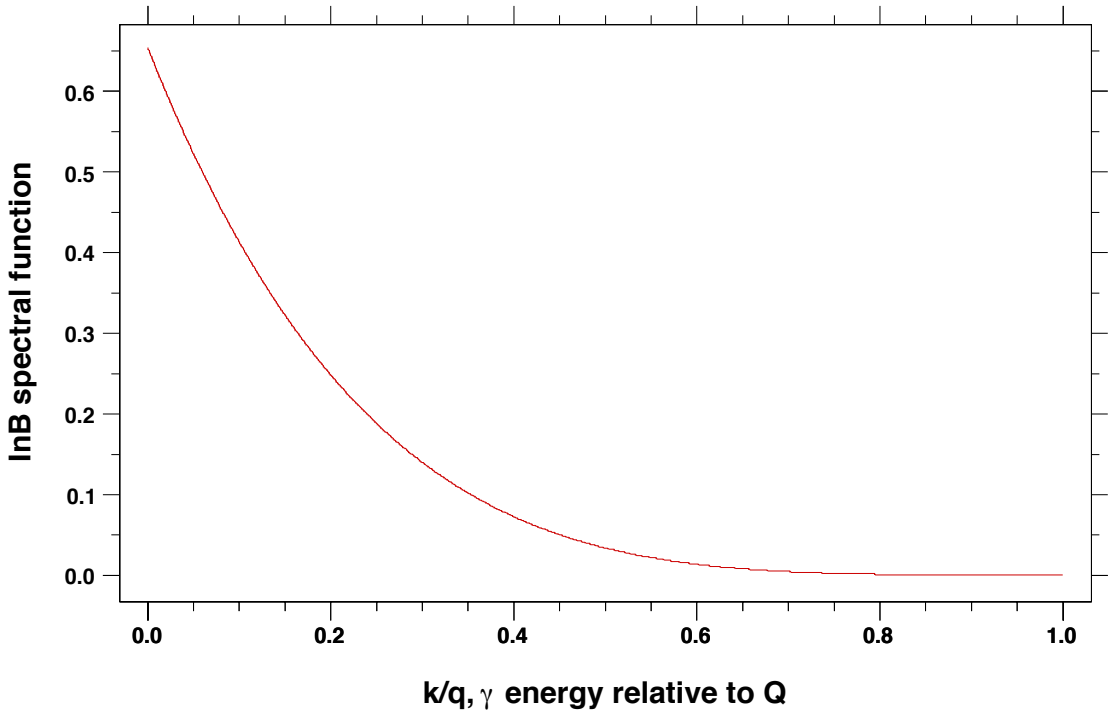

Figure 2: Internal Bremsstrahlung Spectral Function for Tritium Beta Decay

3.3 External Bremsstrahlung

Kramers has calculated the total bremsstrahlung spectrum for a β that is completely stopped by a thick target as (see for example [3]):

$$\frac{d^2N_\gamma}{dN_\beta dk} = \left[\frac{2\alpha}{3\sqrt{3}\ell}\right]\frac{2Z_t}{k}\left[\frac{E-k}{m}\right] \tag{14}$$

where $\ell \approx 4$ for the case of tritium β decay [4], Z_t is the atomic number of the target, and E is the initial kinetic energy of the β. After convoluting this with the β spectrum, we find the external bremsstrahlung spectral function can be written as:

$$S(k) = \frac{m^5}{Q^5}\left(\frac{4\pi Z_t}{3\sqrt{3}\ell}\right)\left[\frac{1}{m^6}\int_{m+k}^{W_0}\left(p+\pi\alpha Z_d W\right)W(W_0-W)^2\left(W-m-k\right)\,dW\right] \tag{15}$$

This integral can be written in closed form as:

$$S(k) = \left(\frac{4\pi Z_t}{3\sqrt{3}\ell}\right)\left[\frac{K_1 + \pi\alpha Z_d K_2}{q^5}\right] \tag{16}$$

$$\begin{aligned} 240K_1 = {} & 105\left(s-u+\xi\right) - \left(25k+120q\right)u + \left(85q+60k\right)s + 60\xi\left(2q+k\right) \\ & \left(+6k^2 - 20qk - 30q^2\right)u + \left(4q^2+40qk\right)s + 30q\xi\left(2k+q\right) + \\ & \left(-2k^3 + 4qk^2 + 10q^2k\right)u - 12q^2ks + \\ & \left(+16k^4 - 56qk^3 + 60q^2k^2\right)u + \left(12q^4 - 32q^3k\right)s + \\ & \left(+8k^5 - 24qk^4 + 20q^2k^3\right)u + \left(4q^5 - 8q^4k\right)s \end{aligned} \tag{17}$$

$$\begin{aligned} 60K_2 = {} & 5q^4 - 20kq^3 + 5k^4 + 30k^2q^2 - 20k^3q + \\ & 4q^5 - 10kq^4 + 6k^5 + 20k^3q^2 - 20k^4q + \\ & q^6 - 2kq^5 + 2k^6 + 5k^4q^2 - 6k^5q \end{aligned} \tag{18}$$

where we've introduced two new unitless parameters:

$$\xi = \log\left(\frac{1+k+u}{1+q+s}\right) \qquad u = \sqrt{2k+k^2} \tag{19}$$

4 X-Ray Dose Rate for the Tritium Target

4.1 Dose Rate Scale

The biological effect on some tissue T due to the radiation dose absorbed by the tissue is given by:

$$d_T = \int \left[\frac{d^2N_r}{dtdE}\right] E\left(\Delta t\right) \exp\left(-\sum_b [N_b]\sigma_b(E)z_b\right)\left[\frac{A_T}{4\pi R_T^2}\right]\left[1 - \exp\left(-[N_T]\sigma_T(E)z_T\right)\right]\frac{Q_r w_T}{m_T}\,dE \tag{20}$$

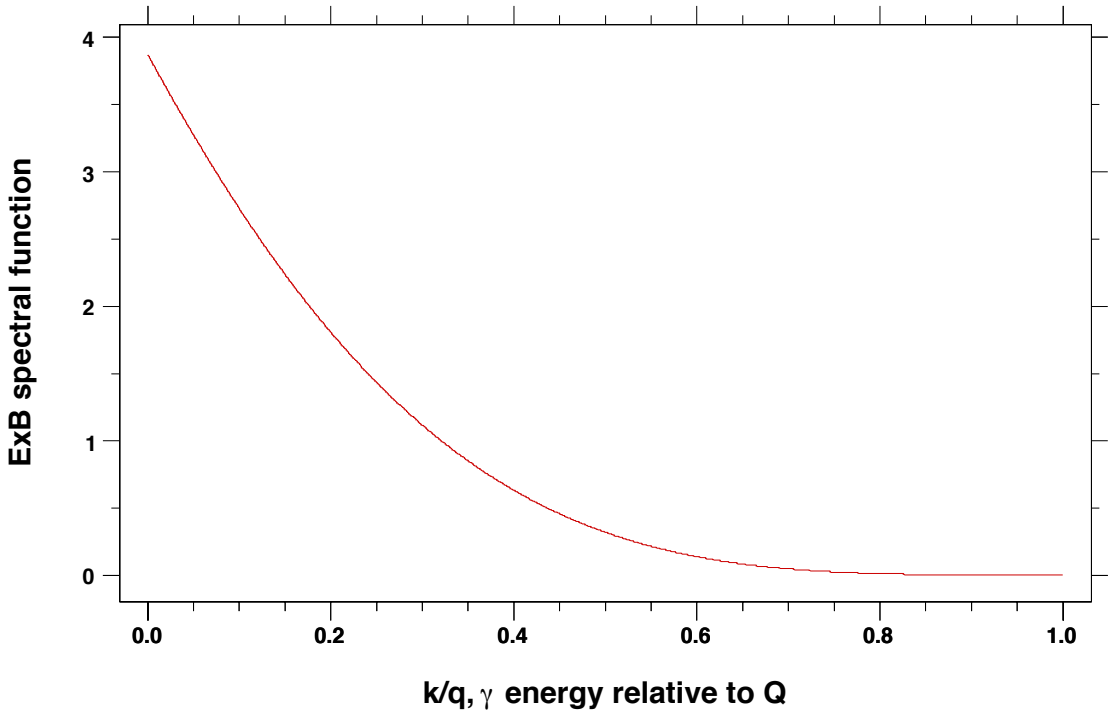

Figure 3: External Bremsstrahlung Spectral Function for Tritium Beta Decay Electron Stopped by Aluminum

where N_r is the number ionizing particles, t is time, E is the energy of the ionizing particle, Δt is the exposure time, $[N_b]$ is the number density of barrier particles that attenuate the radiation, σ_b is the absorption cross section of the barrier, z_b is the thickness of the barrier, A_T is the area of the tissue, R_T is the distance between the source and the tissue, $[N_T]$ is the number density of the tissue, σ_T is the absorption cross of the tissue, z_T is the thickness of the tissue, Q_r is the quality factor of the radiation, w_T is the tissue weight factor, and m_T is the mass of the tissue.

In our case for x-rays, this is written as:

$$d_T = \left[\frac{\alpha Q N_p}{\pi I_0 \tau}\right]\left[\frac{(\Delta t)\Omega_T Q_\gamma w_T q}{4\pi m_T}\right]\underbrace{\int_0^q S(k) f_b(k)(1 - f_T(k))\,\frac{dk}{q}}_{B} \tag{21}$$

where Ω_T is the solid angle subtended by the tissue, Q_γ is the quality factor for x-rays, $f_b(k)$ is the fraction of x-rays that are transmitted by the barriers, and $1 - f_T(k)$ is the

fraction of x-rays absorbed by the tissue. Plugging in $Q_\gamma = 1$, $Q = 18.6$ keV, $q = 0.0364$, $I_0 = 1.57$, $N_p/\tau = 1000$ Ci, and $w_T = 0.2$, we can rewrite this as:

$$\frac{d_T}{\Delta t} = \left[\frac{426 \text{ mrem}}{\text{hr}}\right]\left[\frac{N_p/\tau}{1000 \text{ Ci}}\right]\left[\frac{\Omega_T}{4\pi}\right]\left[\frac{w_T}{0.2}\right]\left[\frac{1 \text{ kg}}{m_T}\right] B \tag{22}$$

where B is the integral underbraced in Eqn. (21). In order to simplify the integral represented by B, we'll define the average fraction of radiation absorbed by the tissue as:

$$1 - \bar{f}_T = \frac{B}{B_0} \qquad B_0 = \int_0^q S(k) \exp\left(-\sum_b [N_b]\sigma_b(E) z_b\right) \frac{dk}{q} \tag{23}$$

where $\bar{f}_T$ is the average fraction of radiation that is transmitted through the tissue and B_0 is calculated assuming that all of the radiation is absorbed by the tissue. With no barriers ($z_b = 0$), there is no attenuation and $B_0 = 0.123$ for InB and $B_0 = 0.860$ for ExB.

4.2 Dose Rate Per Unit Area

At the outer surface of the Al wall, the InB spectrum has been attenuated by the Al and the Kramers formula already accounts for the attenuation of the ExB spectrum. Numerically integrating over the product of the InB spectrum with the photon absorption cross section for Al, we find $B_0 = 4.3 \times 10^{-4}$ for InB, which is heavily attenuated by the Al wall, see left side of Fig. (4). This adds negligibly to the $B_0 = 0.860$ for ExB. If we assume that the x-ray intensity if uniformly distributed over the outer surface area of the target cylinder, then the dose rate per unit area is:

$$\frac{d_T}{(\Delta t) A_T} = \left[\frac{3.6 \text{ mrem}}{\text{hr} \cdot \text{cm}^2}\right]\left[\frac{N_p/\tau}{1000 \text{ Ci}}\right]\left[\frac{w_T}{0.2}\right]\left[\frac{1 \text{ kg}}{m_T}\right](1 - \bar{f}_T) \tag{24}$$

If air separates the person from the target, then the ExB is attenuated using the right side of Fig. (4) to give the results in Tab. (1). Note that we have approximated the target

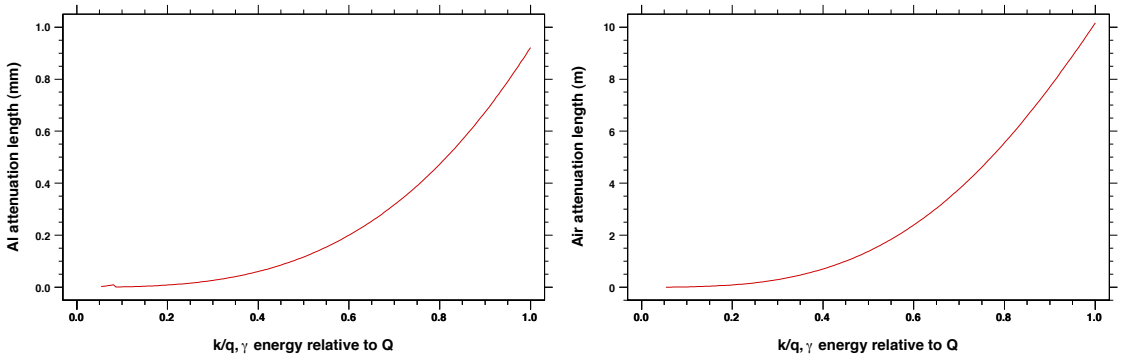

Figure 4: X-Ray Attenuation Length in Solid Aluminum & Dry Air at STP. Note that the target wall thickness is about 0.46 mm. [5]

distance	B_0	$d_t/(\Delta t)/A_T$
surface	0.860	3.6
1 mm	0.815	3.2
3 mm	0.748	2.6
1 cm	0.620	1.4
3 cm	0.474	0.47
10 cm	0.303	0.062
30 cm	0.165	0.0052

Table 1: Dose Rate Per Unit Area As a Function of Distance From Target In Air. The dose rate is given as mrem/hr/cm^2, see Eqn. (24).

cylinder as a sphere of equivalent surface area in order to estimate the reduction in the dose rate due to the smaller subtended solid angle.

References

[1] Coursey J.S. Zucker M.A. Berger, M.J. and J. Chang. (2005), ESTAR, PSTAR, and ASTAR: Computer Programs for Calculating Stopping-Power and Range Tables for Electrons, Protons, and Helium Ions (version 1.2.3). [Online] Available: http://physics.nist.gov/Star [2011-02-22]. *National Institute of Standards and Technology, Gaithersburg, MD*.

[2] R. R. Lewis and G. W. Ford. Coulomb Effects in Inner Bremsstrahlen. *Phys. Rev.*, 107(3):756–765, Aug 1957.

[3] W. A. Jennings. A Theoretical Study of Radiation Outputs and Qualities from a Beryllium Window Tube Operated at Low Kilovoltages (10-50kVp). *Br J Radiol*, 26(304):193–206, 1953.

[4] B. Budick, Jiansheng Chen, and Hong Lin. Bremsstrahlung from tritium β decay. *Phys. Rev. C*, 46(4):1267–1275, Oct 1992.

[5] Hubbell J.H. Seltzer S.M. Chang J. Coursey J.S. Sukumar R. Zucker D.S. Berger, M.J. and K. Olsen. (2010), XCOM: Photon Cross Section Database (version 1.5). [Online] Available: http://physics.nist.gov/xcom [2011-02-22]. *National Institute of Standards and Technology, Gaithersburg, MD*.