Laser Power Requirement of Optical Pumping and Rate Equations

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Abstract

An attempt to calculate the minimum laser power needed for optical pumping. Also some rate equation stuff.

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1 Cross Sections and Rates

A particle passes through a finite target region filled with target points. When the particle encounters a target point, an interaction takes place. The probability that a single interaction occurs p_1 is dependant on the target point density [N], the length of the target region l, and some factor σ that is dependant solely on the nature of interaction:

$$p_1 \;=\; \sigma[\mathrm{N}]l \quad\rightarrow\quad \sigma \;=\; \frac{p_1}{[\mathrm{N}]l} \;=\; \frac{p_1}{\phi} \tag{1}$$

The factor σ is called the interaction cross section and has units of area. It is the probability of a single interaction per unit target point density per unit target region length. A cross section is useful because it isolates the part of the interaction probability that is due to the mechanism of the interaction. An equivalent definition of cross section is the probability of interaction per unit flux, where flux ϕ is the number of target points per unit cross sectional area.

The interaction rate γ is defined as the probability of an single interaction per unit time:

$$\gamma \;=\; \frac{p_1}{t} \;=\; \frac{\sigma[\mathrm{N}]l}{t} \;=\; \sigma v[\mathrm{N}] \tag{2}$$

where v is the relative velocity between particle and the target points. Alternatively, this can rewritten using the flux per unit time $\Phi \equiv \frac{\phi}{t}$:

$$\gamma \;=\; \frac{p_1}{t} \;=\; \frac{\sigma\phi}{t} \;=\; \Phi\sigma \tag{3}$$

Both forms of the interaction rate will be used.

2 Alkali

The equilibrium alkali polarization is calculated by:

$$P_A = \frac{\gamma_{op}}{\gamma_{op} + \gamma} \tag{4}$$

where γ_{op} is the optical pumping rate and γ is the total alkali metal relaxation rate. It is useful to express γ as in terms of γ'_{se}, the spin exchange rate from the noble gas to the alkali metal. These quantities are related through the spin exchange efficiency η which is simply the ratio of spin exchange rate with the noble gas to the total rate of alkali relaxation:

$$\eta = \frac{\gamma'_{se}}{\gamma} \quad \rightarrow \quad \gamma = \frac{\gamma'_{se}}{\eta} \tag{5}$$

Note that this γ'_{se} is not to be confused with γ_{se} which is the spin exchange rate from the alkali metal to the noble gas. These are calculated in the following way:

$$\begin{aligned} \gamma'_{se} &= \langle \sigma v \rangle_{se} \, [\mathrm{N}] &(6)\\ \gamma_{se} &= \langle \sigma v \rangle_{se} \, [\mathrm{A}] &(7) \end{aligned}$$

where $\langle \sigma v \rangle_{se}$ is the velocity averaged spin exchange cross section and [N] & [A] are the noble gas and alkali metal densities.

The optical pumping rate for monochromatic light with a frequency of ν is:

$$\gamma_{op} = \Phi\sigma \tag{8}$$

where Φ is the number of photons per unit cross sectional area per unit time and σ is the absorption cross section. When both the photon flux and the absorption cross section have a frequency dependance, the optical pumping rate is generalized to:

$$\begin{aligned} \gamma_{op} &= \int_0^\infty \Phi(\nu)\sigma(\nu)\mathrm{d}\nu &(9)\\ \Phi(\nu) &= \frac{\text{number of photons}}{\text{unit cross sectional area} \;\times\; \text{unit time} \;\times\; \text{unit frequency interval}} &(10)\\ \sigma(\nu) &= \text{photon absorption cross section at}\nu &(11) \end{aligned}$$

The photon absorption cross section $\sigma(\nu)$ is given by a Lorentzian line shape $L(\nu)$:

$$\begin{aligned} \sigma(\nu) &= \left(\frac{\pi}{2}\Gamma_1\sigma_1\right)\left[\frac{1}{\pi}\frac{\frac{\Gamma_1}{2}}{(\nu-\nu_1)^2+\left(\frac{\Gamma_1}{2}\right)^2}\right] &(12)\\ &= \frac{\pi}{2}\Gamma_1\sigma_1 L(\nu) &(13)\\ L(\nu) &= \frac{1}{\pi}\frac{\frac{\Gamma_1}{2}}{(\nu-\nu_1)^2+\left(\frac{\Gamma_1}{2}\right)^2} &(14)\\ & &(15) \end{aligned}$$

such that:

$$\begin{aligned} \lim_{\left(\frac{\Gamma_1}{\nu_1}\right)\to 0} \int_0^\infty L(\nu)\mathrm{d}\nu &= 1 &(16)\\ \int_0^\infty \sigma(\nu)\mathrm{d}\nu &= \frac{\pi}{2}\Gamma_1\sigma_1 &(17)\\ \sigma(\nu_1) &= \sigma_1 &(18) \end{aligned}$$

Under our conditions, $\frac{\Gamma_1}{\nu_1} = \frac{160 \text{ GHz}}{3.8\times10^5 \text{ GHz}} = 4\times10^{-4} \approx 0$, so the Lorentzian lineshape has unit normalization. It is straight forward to show that the (17) can be related to the oscillator strengh of the transition f by the sum rule (See Merzbacher):

$$\begin{aligned} \int_0^\infty \sigma(\nu)\mathrm{d}\nu &= \pi r_e c f_1 &(19)\\ \pi r_e c f_1 &= \frac{\pi}{2}\Gamma_1\sigma_1 &(20)\\ \sigma_1 &= \frac{2 r_e c f_1}{\Gamma_1} &(21) \end{aligned}$$

The photon flux $\Phi(\nu)$ is given by a Gaussian lineshape $G(\nu)$:

$$\begin{aligned}
\Phi(\nu) &= \frac{\phi_2}{\sigma_2\sqrt{2\pi}}\exp\left(-\frac{(\nu-\nu_2)^2}{2\sigma_2^2}\right) && (22)\\
&= \phi_2 G(\nu) && (23)\\
G(\nu) &= \frac{1}{\sigma_2\sqrt{2\pi}}\exp\left(-\frac{(\nu-\nu_2)^2}{2\sigma_2^2}\right) && (24)
\end{aligned}$$

such that:

$$\begin{aligned}
\lim_{\left(\frac{\sigma_2}{\nu_2}\right)\to 0}\int_0^\infty G(\nu)\mathrm{d}\nu &= 1 && (25)\\
\int_0^\infty \Phi(\nu)\mathrm{d}\nu &= \phi_2 && (26)\\
\Phi(\nu_2) &= \frac{\phi_2}{\sigma_2\sqrt{2\pi}} && (27)
\end{aligned}$$

The photon flux is related to the intensity of the light $I(\nu)$ used for optical pumping:

$$\begin{aligned}
I(\nu) &= h\nu\Phi(\nu) && (28)\\
\lim_{\left(\frac{\sigma_2}{\nu_2}\right)\to 0}\int_0^\infty I(\nu)\mathrm{d}\nu &= I_2 = h\nu_2\phi_2 && (29)\\
I(\nu_2) &= \frac{h\nu_2\phi_2}{\sigma_2\sqrt{2\pi}} && (30)\\
\phi_2 &= \frac{I_2}{h\nu_2} && (31)
\end{aligned}$$

Under our conditions, $\frac{\Gamma_1}{\nu_1} = \frac{430\ \text{GHz}}{3.8\times10^5\ \text{GHz}} = 1.1\times10^{-3} \approx 0$, so the Gaussian lineshape has unit normalization. Putting all this toghether, the optical pumping rate is:

$$\begin{aligned}
\gamma_{op} &= \int_0^\infty \Phi(\nu)\sigma(\nu)\mathrm{d}\nu && (32)\\
&= \int_0^\infty [\phi_2 G(\nu)]\left[\frac{\pi}{2}\Gamma_1\sigma_1 L(\nu)\right]\mathrm{d}\nu && (33)\\
&= \left(\frac{I_2}{h\nu_2}\right)(\pi r_e c f_1)\int_0^\infty G(\nu)L(\nu)\mathrm{d}\nu && (34)\\
&= \left(\frac{I_2\pi r_e c f_1}{h\nu_2}\right)G(\nu_2)\int_0^\infty \frac{G(\nu)}{G(\nu_2)}L(\nu)\mathrm{d}\nu && (35)\\
&= \left(\frac{I_2\pi r_e c f_1 G(\nu_2)}{h\nu_2}\right)\mathcal{L}(\nu_1,\nu_2) && (36)\\
&= \sqrt{\frac{\pi}{2}}\left(\frac{I_2 r_e c f_1}{h\nu_2\sigma_2}\right)\mathcal{L}(\nu_1,\nu_2) && (37)
\end{aligned}$$

where we have defined the unitless lineshape factor $\mathcal{L}$:

$$\begin{aligned}
\mathcal{L}(\nu_1,\nu_2) &= \int_0^\infty \frac{G(\nu)}{G(\nu_2)}L(\nu)\mathrm{d}\nu && (38)\\
\lim_{\left(\frac{\Gamma_1}{\sigma_2}\right)\to 0}\mathcal{L}(\nu_1,\nu_2) &= \frac{1}{G(\nu_2)}\int_0^\infty\left[\lim_{\left(\frac{\Gamma_1}{\sigma_2}\right)\to 0}(G(\nu)L(\nu))\right]\mathrm{d}\nu && (39)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{G(\nu_2)}\int_0^\infty G(\nu)\delta(\nu-\nu_1)\mathrm{d}\nu = \frac{G(\nu_1)}{G(\nu_2)} && (40)\\
&= \exp\left[-\frac{(\nu_1-\nu_2)^2}{2\sigma_2^2}\right] && (41)
\end{aligned}
$$

Under our conditions, $\frac{\Gamma_1}{\sigma_2} = \frac{160 \text{ GHz}}{430 \text{ GHz}} = 0.35 \approx 0$, so the lineshape factor is of order one. Numerically integrating (38) under typical conditions, $\mathcal{L}(\nu_1, \nu_1) \approx 0.87$.

If the beam is gaussian, then ϕ_2 has a radial dependance given by:

$$
\begin{aligned}
\phi_2(r) &= \frac{P_0}{h\nu_2}\frac{2}{w^2\pi}\exp\left(-\frac{2r^2}{w^2}\right) && (42)\\
\int_0^\infty 2\pi r\left(h\nu_2\phi_2(r)\right)\mathrm{d}r &= P_0 && (43)
\end{aligned}
$$

As the laser beam propagates through the cell, the light is attenuated by absorption given by:

$$
\begin{aligned}
\Phi(\nu,r) &= \phi_2(r)G(\nu) = \frac{P_0}{h\nu_2}\left[\frac{2}{w^2\pi}\exp\left(-\frac{2r^2}{w^2}\right)\right]\left[\frac{1}{\sigma_2\sqrt{2\pi}}\exp\left(-\frac{(\nu-\nu_2)^2}{2\sigma_2^2}\right)\right] && (44)\\
\frac{\partial\Phi(\nu,r)}{\partial z} &= -\left[\mathrm{Rb}\right](1-P)\,\sigma(\nu)\Phi(\nu,r) && (45)\\
\int_0^\infty \frac{\partial\Phi(\nu,r)}{\partial z}\mathrm{d}\nu &= -\left[\mathrm{Rb}\right](1-P)\int_0^\infty \sigma(\nu)\Phi(\nu,r)\mathrm{d}\nu && (46)\\
\frac{\partial\phi_2(r)}{\partial z} &= -\left[\mathrm{Rb}\right](1-P)\,\phi_2(r)\int_0^\infty \frac{\pi}{2}\Gamma_1\sigma_1 L(\nu)G(\nu)\mathrm{d}\nu && (47)\\
&= -\left[\mathrm{Rb}\right](1-P)\,\phi_2(r)\frac{\pi}{2}\Gamma_1\sigma_1 G(\nu_2)\mathcal{L}(\nu_1,\nu_2) && (48)\\
&= -\left[\mathrm{Rb}\right](1-P)\sqrt{\frac{\pi}{8}}\frac{\Gamma_1}{\sigma_2}\sigma_1\mathcal{L}(\nu_1,\nu_2)\phi_2(r) && (49)\\
\phi_2(r,z) &= \phi_2(r)\exp\left(-\langle\sigma(\nu)\rangle\left[\mathrm{Rb}\right](1-P)\,z\right) && (50)\\
\langle\sigma(\nu)\rangle &= \sqrt{\frac{\pi}{8}}\frac{\Gamma_1}{\sigma_2}\sigma_1\mathcal{L}(\nu_1,\nu_2) && (51)\\
&= \sqrt{\frac{\pi}{2}}\frac{r_e c f_1}{\sigma_2}\mathcal{L}(\nu_1,\nu_2) && (52)
\end{aligned}
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REFERENCES!!!! & **Diagrams**