Quick Note on Laplace Transform of Multiple Exponential Decays

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Suppose a fluorescence signal $F(t)$ has the following time dependance (t):

$$f(t) = \left[B + \sum_n A_n \exp\left(-\frac{t}{\tau_n} \right) \right] u(t) \tag{1}$$

where B is some background offset, A_n is the ampitude of the n-th exponential with time constant τ_n, and $u(t)$ is the unit step function such that $u(t) = 0$ for $t < 0$ and $u(t) = 1$ for $t > 0$. We can use the fact that a single exponential has the following transform:

$$\mathcal{L}\left\{ \exp\left(-\frac{t}{\tau_n} \right) \right\} = \frac{1}{s + 1/\tau_n} \qquad \text{valid for } \Re\{s\} > -\frac{1}{\tau_n} \tag{2}$$

combined with the fact that Laplace transforms are linear transforms to get:

$$\mathcal{L}\left\{ f(t) \right\} = \frac{B}{s} + \sum_n \frac{A_n}{s + 1/\tau_n} \tag{3}$$

which is valid for $\Re s > 0$ assuming $\tau_n > 0$.

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