Holding Field Angle Measurements

Jaideep Singh, Michael Carl, and Peter A. M. Dolph
University of Virginia
Version 0.10

September 24, 2004

Abstract

This notes describes the procedure used to determine the magnetic field orientation in the vicinity of the cell. Measurements were made on top of the oven. The horizontal angle was measured using rudimentary surveying techniques. The vertical angle was measured by varying the current in the helmholtz coils and noting the vertical angle. Since the measurements were made on top of the oven and not at the center of the pumping chamber of the cell, models are used to estimate the effect of this deviation. An in-depth uncertainty analysis will be presented.

Contents

1 Introduction **1**

2 ϕ_0 measurement **2**

3 Horizontal angle measurement **5**

4 Vertical angle measurement, perpendicular plane **5**

5 Vertical angle measurement, parallel plane **5**

6 Cross product measurement **6**

7 Field magnitude measurement **6**

8 Helmholtz coil model **6**

9 Analysis **6**

A Fitting **6**

1 Introduction

The unprimed coordinate system is defined by the long optics table as depicted in fig. 1. Nominal north (nN) is the direction that you are walking as you exit the lab. The z-axis is directed from the helmholtz coils (HH) towards the long optics table (LOT). The x-axis is directed away from nN or equivalently towards nominal south (nS). The y-axis points up towards the ceiling and is normal to the LOT. The origin of this coordinate system is the nominal northeast corner of the LOT. Various distances (measured by hand) are catalogued in the following figures. The primed coordinate system is defined by the top surface of the oven. The z'-axis is directed from the HH to the LOT along the oven. The x'-axis is directed away from nN or equivalently towards nS along the oven. The y'-axis is the normal to the oven and points away from the

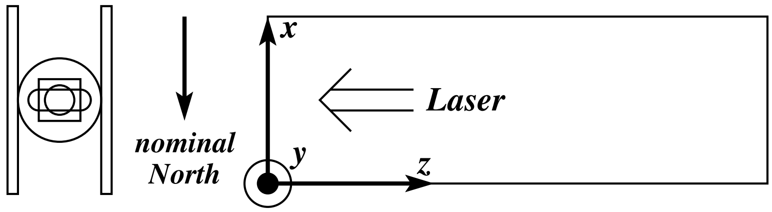

Figure 1: Coordinate system as defined by a top view of the LOT.

oven. The origin of the primed coordinate system is the nominal northeast corner of the oven. Note that corresponding unit vectors of these two coordinate systems might not be parallel:

$$\hat{z} \times \hat{z}' \neq 0 \quad \hat{x} \times \hat{x}' \neq 0 \quad \hat{y} \times \hat{y}' \neq 0 \tag{1}$$

Horizontal and vertical are defined with respect to the y'-axis on top of the oven. The horizontal and vertical components of the HH field as a function of current I are:

$$B_{\rm HH}^{\rm h} = \kappa I \tag{2}$$
$$B_{\rm HH}^{\rm v} = \lambda I \tag{3}$$

What we are calling the Earth's field is actually the field present when $I = 0$. This field is presumably dominated by the Earth's field and therefore will be referred to as such. The vertical and horizontal components of the Earth's field are labeled $B_{\rm E}^{\rm v}$ and $B_{\rm E}^{\rm h}$ respectively. The field used for optical pumping is $\vec{B}_0$ which the vector sum of the Earth's field and the HH field with $I = 3.95 \pm 0.02$ A. Various angles and vectors from fig. 2 are defined in the following way:

$$\vec{B}_{\rm HH}^{\rm v} = \left(\hat{y}' \cdot \vec{B}_{\rm HH}\right) \hat{y}' \tag{4}$$
$$\vec{B}_{\rm HH}^{\rm h} = \vec{B}_{\rm HH} - \vec{B}_{\rm HH}^{\rm v} \tag{5}$$
$$\vec{B}_{\rm E}^{\rm v} = \left(\hat{y}' \cdot \vec{B}_{\rm E}\right) \hat{y}' \tag{6}$$
$$\vec{B}_{\rm E}^{\rm h} = \vec{B}_{\rm E} - \vec{B}_{\rm E}^{\rm v} \tag{7}$$
$$\hat{z}' \cdot \hat{B}_{\rm HH}^{\rm h} = \cos(\phi_0') \tag{8}$$
$$\hat{B}_{\rm E}^{\rm h} \cdot \hat{B}_{\rm HH}^{\rm h} = \cos(\gamma) \tag{9}$$
$$\hat{B}_0^{\rm h} \cdot \hat{B}_{\rm HH}^{\rm h} = \cos(\alpha) \tag{10}$$
$$\hat{B}_0^{\rm h} \cdot \hat{B}_{\rm E}^{\rm h} = \cos(\beta) \tag{11}$$
$$\gamma = \alpha + \beta \tag{12}$$

2 ϕ_0 measurement

The layout for this measurement is given in fig. 3. A compass was aligned to $\vec{B}_0^{\rm h}$ which is denoted by the red line. Markings in 0.5 cm increments were made on a posterboard. This board was placed at three different positions along the z-axis. The marks on the posterboard lie on the x-axis. The angle markings on the compass were aligned with the markings on the posterboard. The relationship between the position of the posterboard marking (z,x) and the angle marking of the compass ϕ with respect to $\hat{B}_0^{\rm h}$ is given by:

$$\tan(\phi + \phi_0) = \frac{x + x_0}{z + z_0} \tag{13}$$

where the position of the compass is ($-z_0$,$-x_0$). The edge of the LOT is in blue. The magenta lines originate from the location of the compass an terminate at the various data points denoted by open circles. The data were fit to the equation given before and the results are also displayed in the layout figure.

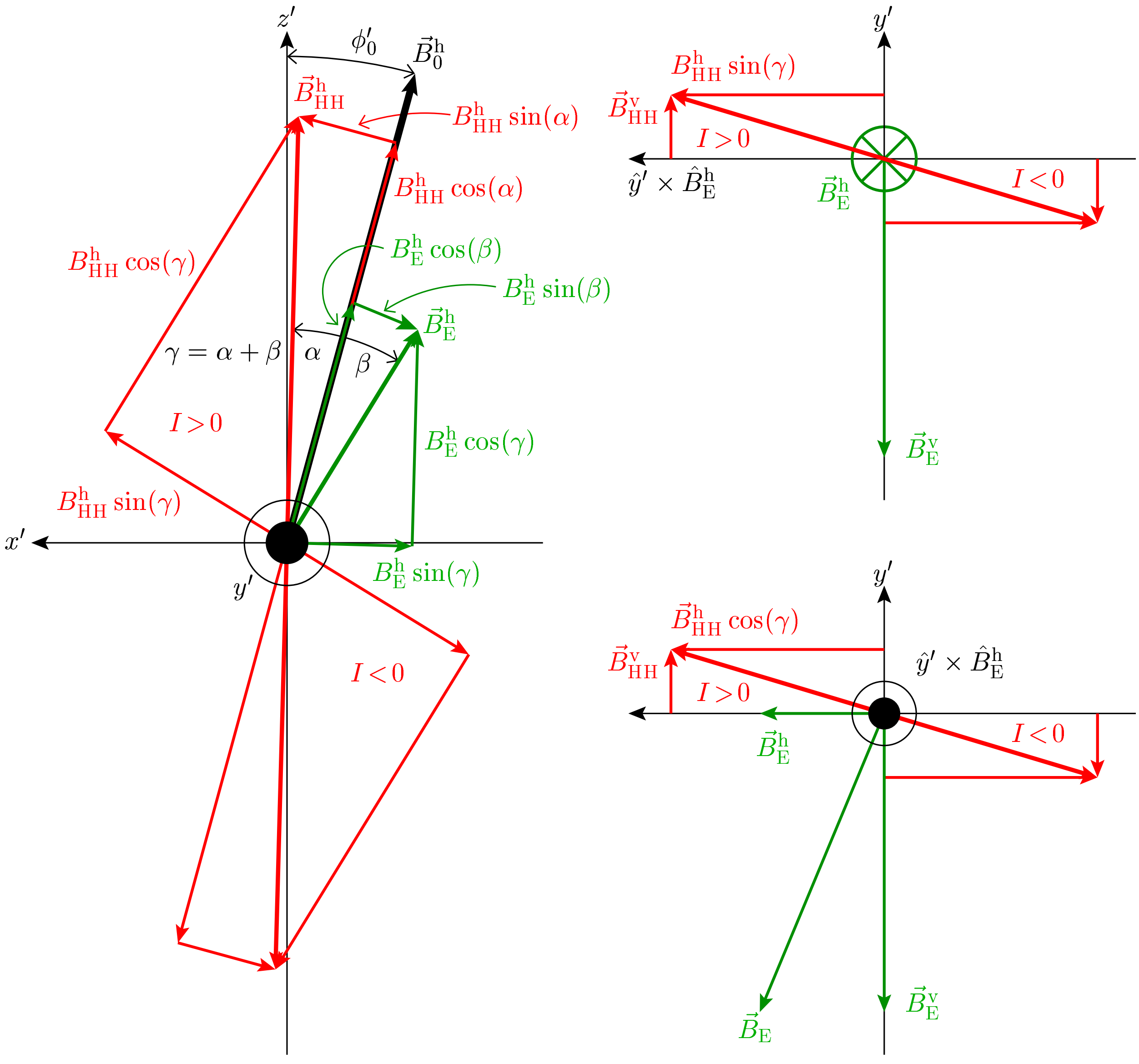

Figure 2: *Left:* Vector components in the z'-x' plane. *Upper right:* Vector components in the plane perpendicular to the Earth's field. *Lower right:* Vector components in the plane parallel to the Earth's field.

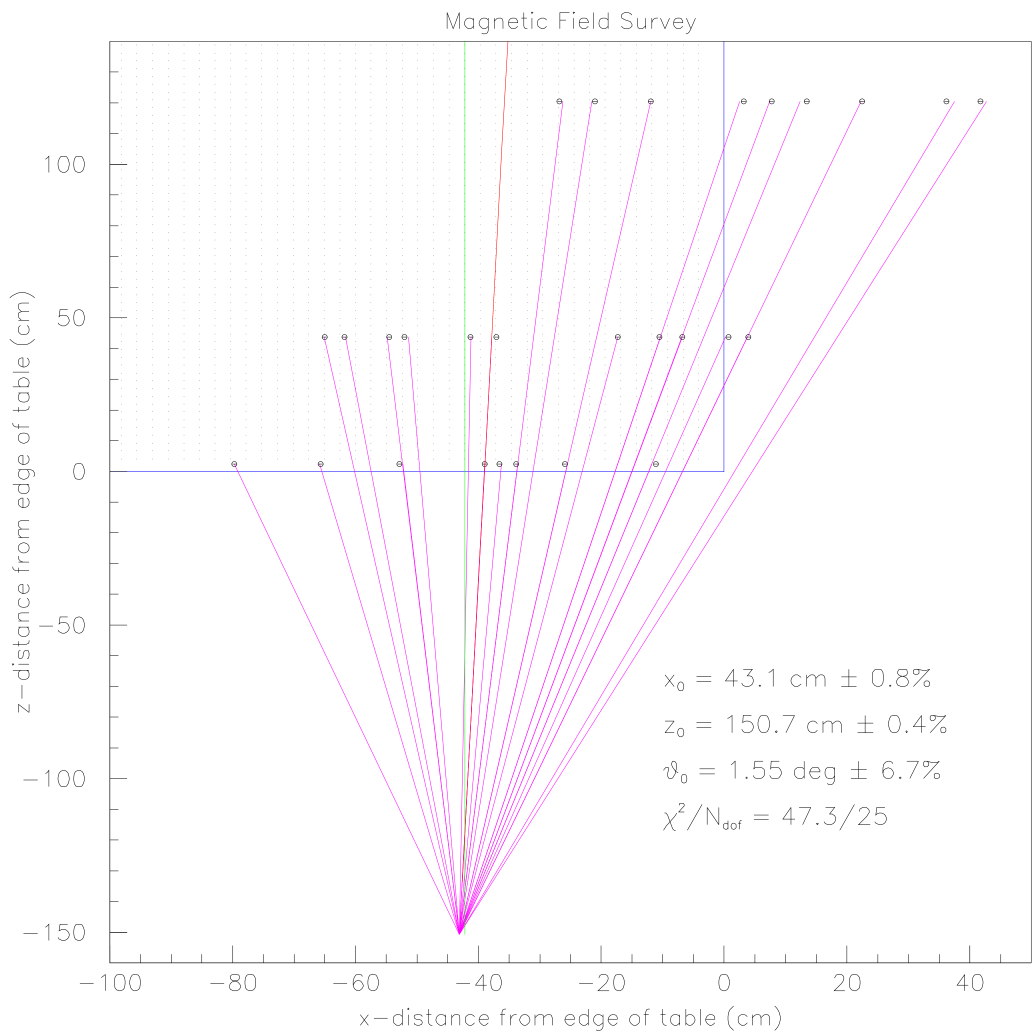

Figure 3: The layout

3 Horizontal angle measurement

Again, the compass is aligned with $\vec{B}_0^{\mathrm{h}}$. In this case, the current is varied and the angle of the new total field with respect to $\vec{B}_0^{\mathrm{h}}$ is recorded. The relationship of current to this angle ϕ is given by:

$$
\begin{aligned}
\phi &= 0 \;\Rightarrow\; \kappa I_0 \sin(\alpha) = B_{\mathrm{E}}^{\mathrm{h}} \sin(\beta) && (14)\\
\tan(\phi) &= \frac{-B_{\mathrm{HH}}^{\mathrm{h}} \sin(\alpha) + B_{\mathrm{E}}^{\mathrm{h}} \sin(\beta)}{B_{\mathrm{HH}}^{\mathrm{h}} \cos(\alpha) + B_{\mathrm{E}}^{\mathrm{h}} \cos(\beta)} && (15)\\
&= \frac{-\kappa I \sin(\alpha) + B_{\mathrm{E}}^{\mathrm{h}} \sin(\beta)}{\kappa I \cos(\alpha) + B_{\mathrm{E}}^{\mathrm{h}} \cos(\beta)} && (16)\\
&= \frac{-\kappa I \sin(\alpha) + \kappa I_0 \sin(\alpha)}{\kappa I \cos(\alpha) + B_{\mathrm{E}}^{\mathrm{h}} \cos(\beta)} && (17)\\
&= \frac{\kappa \sin(\alpha)(I_0 - I)}{\kappa I \cos(\alpha) + B_{\mathrm{E}}^{\mathrm{h}} \cos(\beta)} && (18)\\
&= \frac{I_0 - I}{\frac{\kappa I \cos(\alpha)}{\kappa \sin(\alpha)} + \frac{B_{\mathrm{E}}^{\mathrm{h}} \cos(\beta)}{\kappa \sin(\alpha)}} && (19)\\
&= \frac{I_0 - I}{I \cot(\alpha) + \frac{B_{\mathrm{E}}^{\mathrm{h}} \cos(\beta)}{\frac{B_{\mathrm{E}}^{\mathrm{h}} \sin(\beta)}{I_0}}} && (20)\\
&= \frac{I_0 - I}{I \cot(\alpha) + I_0 \cot(\beta)} && (21)
\end{aligned}
$$

A fit to this equation yields I_0,α,β,γ,and $\frac{B_{\mathrm{E}}^{\mathrm{h}}}{\kappa}$.

4 Vertical angle measurement, perpendicular plane

A dip needle is basically a compass that is mounted vertically. For this measurement, it is orientated such that its needle is perpendicular to the horizontal component of the Earth's field. In this case, the angle measured θ with respect to $\vec{B}_{\mathrm{E}}^{\mathrm{v}}$ as a function of current in the HH is:

$$
\begin{aligned}
\tan(\theta) &= -\frac{B_{\mathrm{HH}}^{\mathrm{h}} \sin(\gamma)}{B_{\mathrm{E}}^{\mathrm{v}} - B_{\mathrm{HH}}^{\mathrm{v}}} && (22)\\
&= -\frac{\kappa I \sin(\gamma)}{B_{\mathrm{E}}^{\mathrm{v}} - \lambda I} && (23)
\end{aligned}
$$

A fit to this equation yields $\frac{B_{\mathrm{E}}^{\mathrm{v}}}{\kappa \sin(\gamma)}$ and $\frac{\lambda}{\kappa \sin(\gamma)}$.

5 Vertical angle measurement, parallel plane

If the dip needle is places in the plane of the Earth's field, then the equation is modified to:

$$
\begin{aligned}
\tan(\theta) &= \frac{-B_{\mathrm{HH}}^{\mathrm{h}} \cos(\gamma) - B_{\mathrm{E}}^{\mathrm{h}}}{B_{\mathrm{E}}^{\mathrm{v}} - B_{\mathrm{HH}}^{\mathrm{v}}} && (24)\\
&= \frac{-\kappa I \cos(\gamma) - B_{\mathrm{E}}^{\mathrm{h}}}{B_{\mathrm{E}}^{\mathrm{v}} - \lambda I} && (25)\\
&= \frac{\kappa I \cos(\gamma) + B_{\mathrm{E}}^{\mathrm{h}}}{\lambda I - B_{\mathrm{E}}^{\mathrm{v}}} && (26)
\end{aligned}
$$

A fit to this equation yields $\frac{B_{\mathrm{E}}^{\mathrm{v}}}{\kappa \cos(\gamma)}$, $\frac{\lambda}{B_{\mathrm{E}}^{\mathrm{v}}}$, and $\frac{B_{\mathrm{E}}^{\mathrm{h}}}{B_{\mathrm{E}}^{\mathrm{v}}}$.

6 Cross product measurement

7 Field magnitude measurement

8 Helmholtz coil model

9 Analysis

The initial guesses for the various parameters are:

$$I_0 = 3.972 \text{ Amps} \quad (27)$$

$$\alpha = 0.50 \text{ deg} \quad (28)$$

$$\beta = 30.00 \text{ deg} \quad (29)$$

$$\gamma = 30.50 \text{ deg} \quad (30)$$

$$\kappa = \frac{12.95}{3.95} \frac{\text{gauss}}{\text{Amp}} \quad (31)$$

$$\lambda = \frac{0.161}{3.95} \frac{\text{gauss}}{\text{Amp}} \quad (32)$$

$$B_{\mathrm{E}}^{\mathrm{h}} = 0.207 \text{ gauss} \quad (33)$$

$$B_{\mathrm{E}}^{\mathrm{v}} = 0.483 \text{ gauss} \quad (34)$$

A Fitting

The following function occurs often in this note:

$$f(x) = \frac{ax+b}{cx+d} \quad (35)$$

where a,b,c, and d are real parameters. It appears that there are four parameters that uniquely determine this funtion. However, on closer inspection, it turns out that only *three* parameters are needed to uniquely determine the function. This can be shown by considering the main features of the functions:

1. zeroes, $f(x_0) = 0$
2. poles, $\lim x \to x_p f(x) \to \infty$
3. asymptotic values, $\lim x \to \pm\infty f(x) = f_{\pm\infty}$

It is easy to see that these are:

$$x_0 = -\frac{b}{a} \qquad x_p = -\frac{d}{c} \qquad f_{\pm\infty} = f_\infty = \frac{a}{c} \quad (36)$$

In fact the original can be rewritten to make this more transparent:

$$f(x) = f_\infty \frac{x - x_0}{x - x_p} \quad (37)$$

Since only three parameters are needed to uniquely determing the function, the fit routine should only have three parameters. If $b = 0$ or equivalenty $x_0 = 0$, then the function can be linearized in the following way:

$$f(x) = f_\infty \frac{x}{x - x_p} \quad (38)$$

$$f^{-1} = f_\infty^{-1} \frac{x - x_p}{x} \quad (39)$$

$$f^{-1} = f_\infty^{-1}\left(1 - x_p x^{-1}\right) \quad (40)$$

$$g(y) = a'y + b' \quad (41)$$

$$g = f^{-1} \qquad y = x^{-1} \qquad a' = -\frac{x_p}{f_\infty} \qquad b' = f_\infty^{-1} \quad (42)$$

If $d = 0$ or equivalently $x_p = 0$, the function can be linearized in the following way:

$$f(x) = f_\infty \frac{x - x_0}{x} \quad (43)$$

$$f(x) = f_\infty \left(1 - \frac{x_0}{x}\right) \quad (44)$$

$$f(y) = a'y + b' \quad (45)$$

$$y = x^{-1} \qquad a' = -f_\infty x_0 \qquad b' = f_\infty \quad (46)$$

Finally, in the case $x_0 = x_p$, the function is simply a constant for all x ($f(x) = f_\infty$) and only one parameter is needed.