A Potpourri of Combiner Fiber Studies

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Abstract

The propagation of the light that is emitted from both the tunable Ti:Sapphire laser (single mode) and the Coherent FAPs (multimode) is found to be modeled very well using gaussian optics. This is useful when trying to obtain a beam of a certain size at some prescribed distance from a converging lens. This technote will describe (1) gaussian optics, (2) how to calculate the beam radius after passing through a thin converging lens, and (3) measurements made to obtain the gaussian beam parameters of the lasers in the lab. describe gaussian beam optics tabulate results from combiner and diffuser tests

Contents

1 Introduction to Gaussian Beam Optics

The theoretical treatment that follows relies heavily on Seigman. It's best to consult Siegman early and often. However, an attempt has been made to make this section as self contained as possible.

1.1 Derivation of the Paraxial Wave Equation

The wave equation for electromagnetic radiation propagating in a source free, linear, uniform, and isotropic medium in SI units is:

$$\mu\epsilon\frac{\partial^2\vec{\mathbf{E}}}{\partial t^2} - \nabla^2\vec{\mathbf{E}} = 0 \tag{1}$$

where μ is the magnetic permeability and ϵ is the electric permittivity. If on a coarse level the radiation is a plane wave, then it is useful to consider solutions of the form:

$$\vec{\mathbf{E}}(t,\vec{r}) = \vec{E}(\vec{r})\underbrace{\exp\left[i\left(\omega t - kz\right)\right]}_{\text{plane wave}} \tag{2}$$

$$k = \frac{2\pi}{\lambda} \quad \text{(wavenumber)} \tag{3}$$

$$\omega = 2\pi\nu \quad \text{(angular frequency)} \tag{4}$$

$$k^2 = \omega^2\mu\epsilon \quad \text{(dispersion relation)} \tag{5}$$

where the plane wave aspect of the radiation is explicitly factored out. The wave equation then becomes:

$$\nabla_t^2\vec{E} + \frac{\partial^2\vec{E}}{\partial z^2} - 2ik\frac{\partial\vec{E}}{\partial z} = 0 \tag{6}$$

where the subscript t refers to the plane transverse to the propagation axis z. The characterisic spread in the distribution of the electromagnetic energy in the transverse plane defines the "size" of the beam. This is ultimately described by the form of $\vec{E}$. If the characteristic variation of the size of the beam as it propagates $\left(\frac{\partial^2\vec{E}}{\partial z^2}\right)$ is small compared to either the characteristic size of the features in the transverse shape of the beam $\left(\nabla_t^2\vec{E}\right)$ or to the oscillations described by the wavelength of the light $\left(2k\frac{\partial\vec{E}}{\partial z}\right)$, then the second derivative term in z can be dropped. In other words, the argument being made is that the $\exp(-ikz)$ term contains the "fast" dependance on z and consequently $\vec{E}$ contains the "slow" dependance on z. This is the paraxial approximation and yields the paraxial wave equation:

$$\nabla_t^2\vec{E} - 2ik\frac{\partial\vec{E}}{\partial z} = 0 \tag{7}$$

Note that the transverse laplacian operator can be evaluated in either rectangular (x, y) or polar (r, θ) coordinates.

1.2 General Form of the Solutions to the Paraxial Wave Equation

From an experimental (practical) viewpoint, useful solutions to the paraxial equation take the form of:

$$\vec{E}(\vec{r}) = \overbrace{\vec{\varepsilon}}^{\text{polarization}} \times \underbrace{E_0\begin{pmatrix} x,y,z \\ r,\theta,z \end{pmatrix}}_{\text{beam shape amplitude}} \times \overbrace{\frac{1}{w(z)}\exp\left(-\frac{r^2}{w^2(z)}\right)}^{\text{gaussian envelope}} \times \underbrace{\exp\left(-i\frac{k}{2}\frac{r^2}{R(z)}\right)}_{\text{spherical like wave}} \tag{8}$$

The gaussian envelope results in a beam of finite extent whose characteristic size is given by a scale factor that depends only on z. This is called beam radius $w(z)$. The spherical wave like phase factor implies that over some region along the propagation axis, the beam appears to be emanating from a source in the form of a spherical wave with radius of curvature $R(z)$. If we define a quantity $q(z)$ called the complex radius of curvature in the following way:

$$\frac{1}{q(z)} \equiv \frac{1}{R(z)} - i\frac{2}{k\omega^2(z)} \tag{9}$$

then the solution can be rewritten as:

$$\vec{E} = \vec{\varepsilon}E_0(\vec{r})\frac{1}{w(z)}\exp\left(-i\frac{k}{2}\frac{r^2}{q(z)}\right) \tag{10}$$

Plugging this test solution into equation (7) yields a differential equation for $E_0(\vec{r})$.

1.3 Exact Solutions to the Paraxial Wave Equation

If the system exhibits rectangular symmetry, then the solutions are the Hermite-gaussian functions where $E_0(\vec{r})$ contains a product of Hermite polynomials:

$$E_0^{nm}(\vec{r}) = \underbrace{\frac{\overbrace{\exp\left[-i(n+m+1)\left(\phi_G(z)-\phi_G(0)\right)\right]}^{\text{Guoy phase term}}}{\sqrt{2^{n+m-1}n!m!\pi}}\frac{1}{w(z)}}_{\text{normalization}} \underbrace{H_n\left(\frac{\sqrt{2}x}{w(z)}\right)H_m\left(\frac{\sqrt{2}y}{w(z)}\right)}_{\text{Hermite polynomials}} \tag{11}$$

where $w(z)$ is the beam radius and $\phi_G(z)$ is called the Guoy phase. The Guoy phase term is related to the Guoy effect which is the relative 180 degree phase shift that a focusing beam picks up after passing through its focal point. If the system exhibits cylindrical symmetry, then the solutions are the Laguerre-gaussian functions where $E_0(\vec{r})$ contains a generalized Laguerre polynomial ($0 \le m \le p$):

$$E_0^{pm}(\vec{r}) = \underbrace{\overbrace{\exp\left[i(2p+m+1)\left(\phi_G(z)-\phi_G(0)\right)\right]}^{\text{Guoy phase term}}\sqrt{\frac{2p!}{(1+\delta_{0m})\,\pi\,(p+m)!}}\frac{1}{w(z)}\left(\frac{\sqrt{2}r}{w(z)}\right)^m}_{\text{normalization}} \overbrace{L_p^m\left(\frac{2r^2}{w^2(z)}\right)}^{\text{Laguerre}}\exp(im\theta) \tag{12}$$

Both sets of functions represent a complete set. Each function within a set is labeled by either nm or pm and is called a spatial mode. A general gaussian beam is defined to be a linear combination of these modes and is consequently an exact solution to the paraxial wave equation. The z dependance of the beam radius w, the complex radius of curvature q, and the Guoy phase ϕ_G are *independant* of mode. Therefore

1.4 Beam Radius $w(z)$ & Radius of Curvature $R(z)$

The equations that govern the propagation of a gaussian beam along the z-axis are:

$$w(z) = w_0\sqrt{1+\left[\frac{(z-z_0)}{z_R}\right]^2} \tag{13}$$

$$R(z) = (z-z_0)\left(1+\left[\frac{z_R}{(z-z_0)}\right]^2\right) \tag{14}$$

At the beam waist, $z = z_0$, the beam radius is a minimum and is equal to $w(z_0) = w_0$. In addition, the radius of curvature is infinity, which implies that the wavefront is a plane. (The size of the beam waist and the location of the beam waist will be interchangeably referred to as the beam waist.) As the beam propagates from the beam waist, the beam radius increases and the radius of curvature decreases. The Rayleigh length z_R defines two zones of behaviour for the gaussian beam. At small distances from the beam waist ($z - z_0 \ll z_R$), the beam radius increases slowly with a soft quadratic dependence. At large distances from the beam waist ($z - z_0 \gg z_R$), the beam radius increases linearly. In the region from the beam waist to the Rayleigh length ($z_0 \leq z \leq z_R$), the radius of curvature quickly decreases from infinity to a minimum of $R(z_R) = 2z_R$. In the region after the Rayleigh length ($z \geq z_R$), the radius of curvature slowly increases from its minimum value back to infinity. If either the beam radius or radius of curvature is given, then the position along the propagation axis is:

$$z(w) = z_0 \pm z_R \sqrt{\left(\frac{w}{w_0}\right)^2 - 1} \tag{15}$$

$$z(R) = z_0 + \frac{R \pm \sqrt{R^2 - 4z_R^2}}{2} \tag{16}$$

The above equations imply that a gaussian beam is symmetric about the beam waist. The beam radius and radius of curvature are related to each other by:

$$w(R) = \sqrt{2}w_0 \left[1 \pm \sqrt{1 - \left(\frac{2z_R}{R}\right)^2}\right]^{-\frac{1}{2}} \tag{17}$$

$$R(w) = \pm z_R \left(\frac{w}{w_0}\right)\left[1 - \left(\frac{w_0}{w}\right)^2\right]^{-\frac{1}{2}} \tag{18}$$

1.5 Divergence & Far Field Behaviour

The instantaneous divergence of a gaussian beam is given by:

$$\psi = \frac{\partial w}{\partial z} = \tan(\theta) = \left(\frac{w_0}{z_R}\right)^2 \frac{z - z_0}{w(z)} \tag{19}$$

$$\psi(w) = \pm\frac{w_0}{z_R}\sqrt{1 - \left(\frac{w_0}{w}\right)^2} \tag{20}$$

$$\psi(z) = \pm\frac{w_0}{z_R}\left(1 + \left[\frac{z_R}{(z - z_0)}\right]^2\right)^{-\frac{1}{2}} \tag{21}$$

where θ is the instantaneous divergence angle. The far field behaviour of a gaussian beam is found by expanding in the $\left(\frac{z-z_0}{z_R}\right) \gg 1$ limit:

$$\psi(z) \simeq \lim_{z\to\infty}\frac{\partial w}{\partial z} = \psi_0 \equiv \frac{w_0}{z_R} = \tan(\theta_0) \tag{22}$$

$$w(z) \simeq \frac{w_0}{z_R}(z - z_0) = \psi_0 (z - z_0) \tag{23}$$

$$R(z) \simeq z - z_0 \tag{24}$$

where we have now defined the far field divergence, ψ_0, explicitly. Rewriting the gaussian beam equations in terms of ψ_0:

$$w(z) = w_0\sqrt{1 + \psi_0^2\left(\frac{z - z_0}{w_0}\right)^2} \tag{25}$$

$$R(z) = (z-z_0)\left[1+\frac{1}{\psi_0^2}\left(\frac{w_0}{z-z_0}\right)^2\right] \tag{26}$$

$$\psi = \psi_0^2\frac{z-z_0}{w(z)} = \pm\psi_0\sqrt{1-\left(\frac{w_0}{w}\right)^2} = \pm\psi_0\left(1+\left[\frac{z_R}{(z-z_0)}\right]^2\right)^{-\frac{1}{2}} \tag{27}$$

1.6 Complex Radius of Curvature, $q(z)$

1.6.1 Different forms of q

To understand how optical elements effect a gaussian beam, it's convenient to define a quantity called the complex radius of curvature, q:

$$\frac{1}{q(z)} \equiv \frac{1}{R(z)} - i\frac{\alpha}{\omega(z)^2} \tag{28}$$

This quantity combines both the beam radius (amplitude information) and radius of curvature (phase information) of a gaussian beam into one complex number. It can be rewritten as:

$$q(z) = \frac{Rw^2}{w^2-i\alpha R} = \frac{Rw^2}{w^2-i\alpha R}\left[\frac{w^2+i\alpha R}{w^2+i\alpha R}\right] = \frac{Rw^2}{w^4+\alpha^2R^2}\left(w^2+i\alpha R\right) \tag{29}$$

$$= \frac{Rw^4}{w^4+\alpha^2R^2} + i\frac{R^2w^2\alpha}{w^4+\alpha^2R^2} = \frac{R}{1+\frac{\alpha^2R^2}{w^4}} + i\frac{\frac{(z-z_0)^2w^4}{\left(w^2-w_0^2\right)^2}w^2\alpha}{w^4+\frac{\alpha^2(z-z_0)^2w^4}{\left(w^2-w_0^2\right)^2}} \tag{30}$$

$$= \frac{(z-z_0)\frac{w^2}{w^2-w_0^2}}{1+\frac{\alpha^2(z-z_0)^2}{\left(w^2-w_0^2\right)^2}} + i\frac{(z-z_0)^2w^6\alpha}{w^4\alpha^2(z-z_0)^2+w^4\left(w^2-w_0^2\right)^2} \tag{31}$$

$$= \frac{(z-z_0)w^2}{w^2-w_0^2+\frac{\alpha^2(z-z_0)^2}{\left(w^2-w_0^2\right)}} + i\frac{(z-z_0)^2w^2\alpha}{\alpha^2(z-z_0)^2+\left(w^2-w_0^2\right)^2} \tag{32}$$

$$= \frac{(z-z_0)w^2}{w^2-w_0^2+w_0^2} + i\frac{(z-z_0)^2w^2\alpha}{\alpha^2(z-z_0)^2+\frac{\alpha^4(z-z_0)^4}{w_0^4}} \tag{33}$$

$$= (z-z_0) + \frac{i}{\alpha}\frac{w^2}{1+\frac{\alpha^2(z-z_0)^2}{w_0^4}} = (z-z_0) + \frac{i}{\alpha}\frac{w^2}{\frac{w^2}{w_0^2}} \tag{34}$$

$$q(z) = (z-z_0) + i\frac{w_0^2}{\alpha} = (z-z_0) + i\frac{w_0}{\psi_0} = (z-z_0) + iz_R \tag{35}$$

where we have made frequent use of a rearranged form of the beam radius equation:

$$\psi_0^2(z-z_0)^2 = \frac{\alpha^2(z-z_0)^2}{w_0^2} = \left(w^2-w_0^2\right) \tag{36}$$

1.6.2 Components of q

The real part of q is the position relative to the location of the beam waist, $z-z_0$. The imaginary part of q is simply the Rayleigh length, z_R. The magnitude of q is:

$$|q(z)| = \sqrt{(z-z_0)^2+\frac{w_0^4}{\alpha^2}} = \frac{w_0^2}{\alpha}\sqrt{1+\frac{\alpha^2(z-z_0)^2}{w_0^4}} \tag{37}$$

$$|q(z)| = \frac{w_0}{\alpha}w = \frac{z_R}{w_0}w = \frac{1}{\psi_0}w = \left(\frac{w_0}{w}\right)\cdot\mathcal{IM}\ q \tag{38}$$

which is solely a function of the beam radius, w. On the other hand, the phase of q is:

$$\arg q(z) = \tan^{-1}\left(\frac{z_R}{z-z_0}\right) = \tan^{-1}\left(\sqrt{\frac{R-(z-z_0)}{z-z_0}}\right) \tag{39}$$

which is solely a function of the radius of curvature of the beam, R.

1.7 Fundemental Gaussian Beam Mode

The fundamental modes of the Hermite-gaussian and Laguerre-gaussian functions are:

$$E_0^{H00}(\vec{r}) = \exp\left[-i\left(\phi_G(z)-\phi_G(0)\right)\right]\sqrt{\frac{2}{\pi}}\frac{1}{w(z)} \tag{40}$$

$$E_0^{L00}(\vec{r}) = \exp\left[i\left(\phi_G(z)-\phi_G(0)\right)\right]\sqrt{\frac{1}{\pi}}\frac{1}{w(z)} \tag{41}$$

which have similar forms.

$$\vec{E} = \underbrace{\vec{\varepsilon}\exp\left(-i\frac{\pi}{\lambda}\frac{r^2}{R(z)}\right)}_{\text{spherical wave}}\underbrace{E_0(z)\exp\left(-\frac{r^2}{w^2(z)}\right)}_{\text{gaussian}} \tag{42}$$

$$\vec{H} = \sqrt{\frac{\epsilon}{\mu}}\hat{k}\times\vec{E} \tag{43}$$

where $\vec{\varepsilon}$ is the polarization vector and r is the radial distance from the z-axis. Note that a gaussian beam exhibits cylindrical symmetry. Quantities that are explicitly written as a function of z are E_0, the magnitude of the electic field on the propagation axis ($r=0$), R, the radius of curvature of the spherical wave, and w, the beam radius. The intensity is given by the time averaged magnitude of the Poynting vector:

$$I(r,z) = \left|\left\langle\vec{S}\right\rangle\right| = \frac{1}{2}\left|\vec{E}\times\vec{H}^*\right| = \underbrace{\left(\frac{1}{2}\sqrt{\frac{\epsilon}{\mu}}E_0^2(z)\right)}_{\text{normalization}}\underbrace{\exp\left(-2\frac{r^2}{w^2(z)}\right)}_{\text{gaussian}} \tag{44}$$

Not surprisingly, the intensity profile of a gaussian beam in the plane perpendicular to the propagation axis is gaussian. Integrating the intensity over the area contained by a radius r in the perpendicular plane along some point z on the propagation axis yields the power:

$$P(r,z) = \int_0^r 2\pi r' I(r',z)\,dr' = \pi\sqrt{\frac{\epsilon}{\mu}}E_0^2(z)\int_0^r r'\exp\left(-2\frac{r'^2}{w^2(z)}\right)\,dr' \tag{45}$$

$$= \frac{\pi}{4}\sqrt{\frac{\epsilon}{\mu}}E_0^2(z)w^2(z)\left[1-\exp\left(-2\frac{r^2}{w^2(z)}\right)\right] \tag{46}$$

$$P(z) = \lim_{r\to\infty}P(r,z) = \frac{\pi}{4}\sqrt{\frac{\epsilon}{\mu}}E_0^2(z)w^2(z) = P_0 \tag{47}$$

The conservation of energy imposes the constraint that the total power passing through any entire plane perpendicular to the propagation axis must be constant. This allows us to express the magnitude of the electric field and intensity in terms of the total power P_0:

$$P_0 = \frac{\pi}{4}\sqrt{\frac{\epsilon}{\mu}}E_0^2(z)w^2(z) = \frac{\pi}{2}I_0(z)w^2(z) \tag{48}$$

$$E_0(z) = \frac{2}{w(z)}\sqrt{\frac{P_0}{\pi}\sqrt{\frac{\mu}{\epsilon}}} = \sqrt{2I_0(z)\sqrt{\frac{\mu}{\epsilon}}} \tag{49}$$

$$E(r,z) = E_0(z)\exp\left(-\frac{r^2}{w^2(z)}\right) \quad (50)$$

$$I_0(z) = \frac{2}{\pi}\frac{P_0}{w^2(z)} = \frac{1}{2}\sqrt{\frac{\epsilon}{\mu}}E_0^2(z) \quad (51)$$

$$I(r,z) = I_0(z)\exp\left(-2\frac{r^2}{w^2(z)}\right) \quad (52)$$

In this context, $I_0(z)$ represents the peak radial intensity of a gaussian beam. It is natural to characterize the spatial extent of the beam spot with the beam radius $w(z)$. One beam radius, $r = w(z)$, contains $1 - \frac{1}{e^2} \simeq 0.8647$ of total power of the beam. The free propagation of a gaussian beam is uniquely determined given the beam radius $w(z)$ and the radius of curvature $R(z)$.

1.8 Gaussian Beam Parameters, Monochromaticity, & Invariants

To uniquely determine the propagation of a freely propagating gaussian beam, one needs to know the following three gaussian beam parameters:

1. The location of the beam waist, z_0.
2. The beam waist radius, w_0.
3. The Rayleigh length, z_R, or equivalently the far field divergence, $\psi_0 = \frac{w_0}{z_R}$.

An additional gaussian beam parameter that we have not discussed yet is related to the wavelength of the light: $\alpha = M^2\frac{\lambda}{\pi}$. The M^2 factor quantifies the deviation of the beam from an ideal gaussian beam. These deviations can be both from the fact that the light is not purely monochromatic and from the fact that the true intensity distribution is actually a superposition of higher order spatial modes. If the light is perfectly monochromatic and is a purely zeroth order mode, $M^2 = 1$ and $\alpha = \frac{\lambda}{\pi}$. For broadband light coupled to an optical fiber, $M^2 \gg 1$. We have found that the broadband lasers that we use can be accurately modelled as gaussian beams using an appropriate value of $M^2 (\approx 300)$. However, the value of M^2 has to be determined empircally. Note that the M^2 factor is a fudge factor needed to make the equations "work" for the laser beams we use. Whether the M^2 factor "works" as advertised also needs to be empirically determined. Since α is an intrinsic property of the light source, it is constant everywhere at every point along propagation axis. Certain combinations of the gaussian beam parameters remain constant as the beam passes through "passive" optical elements such as lens. (Define "passive") These invariants are:

1. $\alpha = M^2\frac{\lambda}{\pi} = \psi_0 w_0 = \frac{w_0^2}{z_R} = z_R\psi_0^2$, intrinsic to the light source.
2. $1 = \frac{z_R\psi_0}{w_0}$, by definition.

Note that different optical fibers connected to the same laser can cause a change in α. Therefore, a certain combination of fiber and laser is likely to have a unique value of α. Note that α can take the place of z_R and ψ_0 as the third gaussian beam parameter. The relationships among the gaussian beam parameters are summarized here:

$$\alpha = \frac{w_0^2}{z_R} = w_0\psi_0 = z_R\psi_0^2 \quad (53)$$

$$w_0 = \psi_0 z_R = \frac{\alpha}{\psi_0} = \sqrt{\alpha z_R} \quad (54)$$

$$\psi_0 = \frac{w_0}{z_R} = \frac{\alpha}{w_0} = \sqrt{\frac{\alpha}{z_R}} \quad (55)$$

$$z_R = \frac{w_0^2}{\alpha} = \frac{w_0}{\psi_0} = \frac{\alpha}{\psi_0^2} \quad (56)$$

Note that there is an inverse relationship between the far field divergence and the beam waist. This means that a well collimated (low divergence) beam must be large. On the other hand, to focus the beam into a very small spot (small beam waist), the divergence must be large.

2 Thin Lens

2.1 Solving for the Final Beam Parameters

A thin lens with focal length f simply changes the radius of curvature of a gaussian beam. A converging lens has $f > 0$ and a diverging lens has $f < 0$. The transformation equation is remarkably simple and is analogous to the result from ray optics:

$$\frac{1}{q_2} = \frac{1}{q_1} - \frac{1}{f} \tag{57}$$

where q_2 is the the complex radius of curvature of the gaussian beam propagating away from the lens and q_1 is complex radius of curvature of the gaussian beam propagating towards the lens. Note that this equation is evaluated at the z-position of the lens and assumes that the beam propagates along the positive z direction. For simplicity, we'll place the lens at $z = 0$. This could be solved using the first form of q:

$$\frac{1}{R_2(0)} = \frac{1}{R_1(0)} - \frac{1}{f} \tag{58}$$

$$w_2(0) = w_1(0) \tag{59}$$

However, its much simpler to use the second form of $q\,(= (z - z_0) + iz_R)$:

$$\frac{1}{q_2} = \frac{1}{q_1} - \frac{1}{f} \tag{60}$$

$$q_2 = f\frac{q_1}{f - q_1} = f\frac{q_1}{f - q_1}\frac{f - \bar{q_1}}{f - \bar{q_1}} \tag{61}$$

$$= f\frac{-|q_1|^2 + fq_1}{|q_1|^2 + f^2 - f\,(q_1 + \bar{q_1})} \tag{62}$$

$$q_2 = \frac{f}{|q_1|^2 + f^2 - 2f \cdot \mathcal{RE}\ q_1}\left(-|q_1|^2 + fq_1\right) \tag{63}$$

$$\mathcal{RE}\ q_2 = \frac{f}{|q_1|^2 + f^2 - 2f \cdot \mathcal{RE}\ q_1}\left(-|q_1|^2 + f \cdot \mathcal{RE}\ q_1\right) \tag{64}$$

$$\mathcal{IM}\ q_2 = \frac{f}{|q_1|^2 + f^2 - 2f \cdot \mathcal{RE}\ q_1}\left(f \cdot \mathcal{IM}\ q_1\right) \tag{65}$$

Evaluating the real part at the lens, $z = 0$:

$$\mathcal{RE}\ q_2(0) = -z_{02} = \frac{f}{\frac{w_1(0)^2}{\psi_{01}^2} + f^2 + 2fz_{01}}\left(-\frac{w_1(0)^2}{\psi_{01}^2} - fz_{01}\right) \tag{66}$$

$$w_1(0)^2 = w_{01}^2 + \psi_{01}^2 z_{01}^2 \tag{67}$$

$$z_{02} = \frac{f}{\frac{w_{01}^2 + \psi_{01}^2 z_{01}^2}{\psi_{01}^2} + f^2 + 2fz_{01}}\left(\frac{w_{01}^2 + \psi_{01}^2 z_{01}^2}{\psi_{01}^2} + fz_{01}\right) \tag{68}$$

$$= \frac{f}{\frac{w_{01}^2}{\psi_{01}^2} + f^2 + 2fz_{01} + z_{01}^2}\left(\frac{w_{01}^2}{\psi_{01}^2} + z_{01}^2 + fz_{01}\right) \tag{69}$$

$$z_{02} = f\left[\frac{z_{01}\psi_{01}^2\,(f + z_{01}) + w_{01}^2}{\psi_{01}^2\,(f + z_{01})^2 + w_{01}^2}\right] \tag{70}$$

Evaluating the imaginary part at the lens, $z = 0$:

$$\mathcal{IM}\ q_2(0) = z_{R2} = \frac{f}{|q_1|^2 + f^2 - 2f \cdot \mathcal{RE}\ q_1}\left(f \cdot \mathcal{IM}\ q_1\right) \tag{71}$$

$$= \frac{f\psi_{01}^2}{\psi_{01}^2\,(f + z_{01})^2 + w_{01}^2} fz_{R1} \tag{72}$$

$$\frac{z_{R2}}{z_{R1}} = \frac{w_{02}^2}{\alpha}\frac{\alpha}{w_{01}^2} = \frac{f^2\psi_{01}^2}{\psi_{01}^2\left(f+z_{01}\right)^2 + w_{01}^2} \tag{73}$$

$$w_{02}^2 = \frac{f^2\psi_{01}^2 w_{01}^2}{\psi_{01}^2\left(f+z_{01}\right)^2 + w_{01}^2} \tag{74}$$

$$w_{02} = \frac{f\psi_{01}w_{01}}{\sqrt{\psi_{01}^2\left(f+z_{01}\right)^2 + w_{01}^2}} \tag{75}$$

In summary, beam 1 is transformed into beam 2 by a thin converging lens of focal length f:

$$z_{01} \rightarrow z_{02} = m^2 z_{01}\left[\left(1+\frac{z_{01}}{f}\right) + \frac{1}{\psi_{01}^2}\left(\frac{w_{01}^2}{z_{01}f}\right)\right] = m^2 z_{01}\left[\left(1+\frac{z_{01}}{f}\right) + \left(\frac{w_{01}^4}{z_{01}\alpha^2 f}\right)\right] \tag{76}$$

$$w_{01} \rightarrow w_{02} = m w_{01} \tag{77}$$

$$z_{R1} \rightarrow z_{R2} = m^2 z_{R1} \tag{78}$$

$$\psi_{01} \rightarrow \psi_{02} = \frac{\psi_{01}}{m} \tag{79}$$

where, with the chosen convention, $z_{01} < 0$ because the initial gaussian beam is behind the lens. Note the z_{02} might be positive or negative depending on the magnitude of z_{01} relative to f. In addition, m is a postive unitless magnification factor defined as:

$$m \equiv \left[\left(1+\frac{z_{01}}{f}\right)^2 + \frac{1}{\psi_{01}^2}\left(\frac{w_{01}}{f}\right)^2\right]^{-\frac{1}{2}} = \left[\left(1+\frac{z_{01}}{f}\right)^2 + \left(\frac{w_{01}^2}{\alpha f}\right)^2\right]^{-\frac{1}{2}} = \left[\left(1+\frac{z_{01}}{f}\right)^2 + \left(\frac{z_{R1}}{f}\right)^2\right]^{-\frac{1}{2}} \tag{80}$$

Note that it is possible for the beam waist of the outgoing beam to be on the same side of the lens as the beam waist of the incoming beam, namely $z_{01} < 0$ & $z_{02} < 0$. The outgoing beam waist is therefore referred to as virtual. Under typical conditions when using the FAPS, this only happens when lens to fiber distance is less than the focal length of the lens.

2.2 Solving for the Initial Beam Parameters

If the final beam parameters are known and the initial beam parameters are desired, then we solve for q_1:

$$\frac{1}{q_2} = \frac{1}{q_1} - \frac{1}{f} \quad \rightarrow \quad \frac{1}{q_1} = \frac{1}{q_2} + \frac{1}{f} \tag{81}$$

To solve this equation, it is sufficient to swap the subscripts, $1 \leftrightarrow 2$, and make the substitution $f \rightarrow -f$:

$$z_{02} \rightarrow z_{01} = m^2 z_{02}\left[\left(1-\frac{z_{02}}{f}\right) - \frac{1}{\psi_{02}^2}\left(\frac{w_{02}^2}{z_{02}f}\right)\right] \tag{82}$$

$$w_{02} \rightarrow w_{01} = m w_{02} \tag{83}$$

$$z_{R2} \rightarrow z_{R1} = m^2 z_{R2} \tag{84}$$

$$\psi_{02} \rightarrow \psi_{01} = \frac{\psi_{02}}{m} \tag{85}$$

$$m = \left[\left(1-\frac{z_{02}}{f}\right)^2 + \frac{1}{\psi_{02}^2}\left(\frac{w_{02}}{f}\right)^2\right]^{-\frac{1}{2}} \tag{86}$$

where, with the chosen convention, $z_{01} < 0$ because the initial gaussian beam is behind the lens.

2.3 Special Case of Beam Waist at Focal Length

For the special case of either the initial or the final beam waist is located at the focal length, then m is a maximum, the other beam waist is at the focal length on the other side of the lens, and the initial and final beam parameters have the following relationships:

$$m = \frac{\psi_{01}}{w_{01}} f = \frac{\alpha}{w_{01}^2} f = \frac{f}{z_{R1}} \tag{87}$$

$$-z_{01} = z_{02} = f \tag{88}$$

$$\frac{w_{01}}{\psi_{02}} = \frac{w_{02}}{\psi_{01}} = f \tag{89}$$

$$z_{R1} z_{R2} = f^2 \tag{90}$$

3 Gaussian Beam Tests

3.1 Power vs. Radius

To verify that the beam profile has a gaussian intensity distribution, the total power as a function of iris diameter was measured. An iris was centered on the beam exiting the fiber. No lens was used for this test. The iris diameter was varied and the total power contained within the iris aperture was measured using a power meter (TPM 300). The iris diameter was measured using two different calipers by two different people. Each combination of person and caliper was cycled through while taking these measurements to suppress some systematic uncertainties. The power was measured before and after the iris diameter was measured. This was done to account for any change in size to the iris aperture due to the pressure exerted on the iris by the calipers. For obvious reasons, the iris diameter was measured with the laser off. The power as a function of iris radius was made concurrently at two different currents using FAP 4157. The data was fit to equation (46) with two free parameters, P_0 and $w(z)$. As can be seen in figure (1), the data clearly suggest the beam is gaussian.

3.2 Measuring Gaussian Beam Parameters

An iris and a power meter were used to measure the three gaussian beam parameters:

1. w_0, the radius of the beam waist
2. z_0, the location of the beam waist
3. ϕ_0, the far field divergence

First the total power of the light is measured. This is best accomplished by placing the power meter as close to the light source as possible. This insures that most of the beam fits comfortably inside the power meter and therefore very little of the power is lost. Afterwards, the power meter is placed far away on the axis of propagation. Then an iris is placed somewhere in between the light source and power meter. Care was taken to insure that the iris and power meter are properly centered and aligned with respect to the incoming beam.

The iris is contracted until about 86.47% of the total power is measured. The laser is then turned off, and the iris diameter is measured *in situ*. Care was taken to make sure that the iris diameter was not increased due to the pressure of the calipers. Two different people used two different calipers. Each of these four possibilities was cycled through. This measurement was performed at various distances from the lens along the beam propagation axis. Once the iris diameter is measured, the laser is turned on and the power is remeasured. If the iris diameter has significantly changed in size due to interaction with the calipers, then the power will be different then the initial power measurement. The data is fit to equation (13), which relates beam radius to distance from beam waist.

It is possible that the Rayleigh length of the gaussian beam is very small, such as the light emitted from a fiber. This will result in a poor determination of the beam waist radius from the fit. To get around this, it is useful to use a converging lens to roughly collimate the beam to some convenient beam radius. The lens slows the divergence of the beam. Since a gaussian beam is symmetric about the beam waist, the beam waist location can be adequately constrained by the fit if an equal number of data points are taken about the beam waist. The far field divergence can be constrained if enough data points are taken at a distance that is sufficiently far from the beam waist. This corresponds to taking data from one Rayleigh length upto

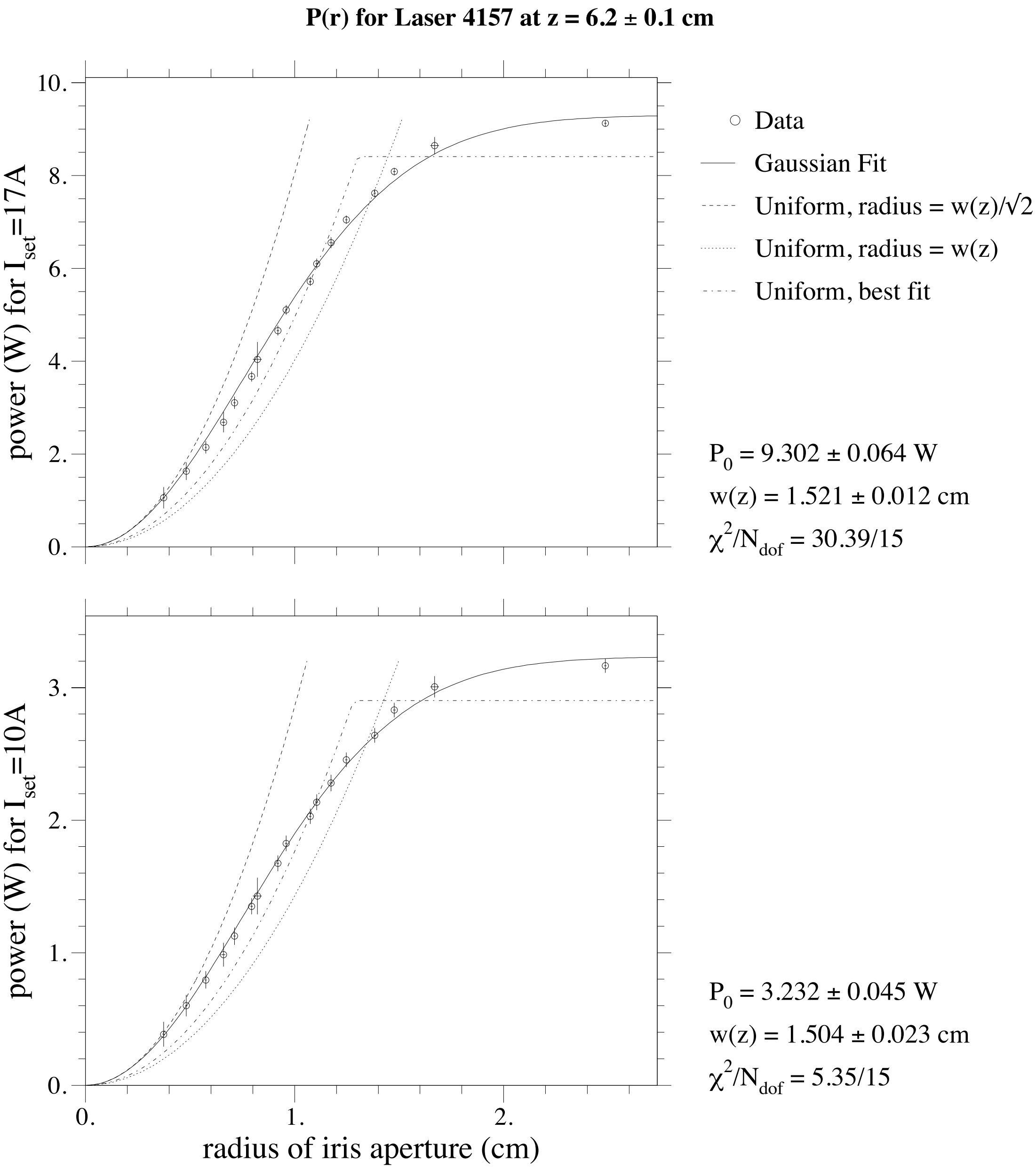


Figure 1: The solid line is a fit to the data assuming a gaussian intensity distribution. The dashed and dotted lines are the curves expected for a uniform intensity distribution with two different radii. The dashed-dotted line is a fit to the data assuming a uniform intensity. The uncertainty bars are statistical and cover a range of $\pm 1\sigma$. Uncertainties from the iris diameter measurements were propagated into uncertainties in the power measured. The uncertainty in the iris radius was taken to be roughly a factor of 3.0 times the average marker resolution of the calipers. The uncertainty in the power measurement was taken to be a combination of the 0.35% rms intrinsic stability of the laser and the quoted 0.5% full scale power meter display uncertainty. Additional uncertainties were added when two measurements of the same quantity were significantly ($> 2\sigma$) different from each other. Although identical uncertainty analyses were performed for the two data sets, the reduced χ^2 for the low power data set is significantly lower than for the high power data set. Either the statistical uncertainties were underestimated for the high power data set or the statistical uncertainties were overestimated for the low power data set. Regardless, the data clearly suggest that the beam is gaussian.

at least two Rayleigh lengths from the beam waist. Because this is relatively easy to do, the fit usually gives reliable values for the far field divergence and the beam waist location. On the other hand, determining the beam waist size reliably is difficult. For best results, the number of points taken within one Rayleigh length from the beam waist should equal the number of points in the region outside one Rayleigh length from the beam waist. In practice this is difficult because the spacing of measurements along the propagation axis becomes prohibitively small.

When using a lens to slow the divergence, the fit only gives the gaussian beam parameters after the lens. To get the gaussian beam parameters before the lens, the fit parameters are used to propagate the beam back through the lens. This is easily done using equations (82) to (86). Note that the lens to fiber distance should be $\geq f$, which insures that the outgoing beam waist is real. If the outgoing beam waist is virtual, then the fit will result in very unreliable values for the beam waist size and location.

At different distances, the data is sensitive to different gaussian beam parameters. Near the beam waist, if it is real, the data is sensitive to the beam waist radius and beam waist location. Far from the beam waist, real or virtual, the data is sensitive to the far field divergence. Therefore it is important to have an adequate number of data points that probe both regions of interest.

In the event that a power meter is not available, we have found that it is sufficient to contract the iris until a small ring of light circles the edge of the iris aperture to determine the beam radius. This technique has worked surprisingly well when the power meter was not available. However, it is not recommened when the beam waist is very small compared to the iris.

3.3 Testing the Lens Transformation Equations

The propagation of the beam from a FAP is adequately described by the gaussian beam equations. This was verified by using a second lens to transform the beam a second time. Note that the first lens, which was located near the fiber, was used to slow the divergence of the beam as described in the previous section. The gaussian beam parameters of the gaussian beam before and after the second lens were measured. The fitted gaussian beam parameters of the beam going into the second lens were propagated forward. These values were compared to the fitted gaussian beam parameters of the beam exiting the second lens. This process was also done in reverse. The results are displayed in figure (2).

Given only these two data sets and this analysis, the following conclusions are made. Gaussian beam optics does a reasonable job giving a qualitative description of the propagation of the beam. At a quantitative level, gaussian beam optics provide adequate guidance for a coarse determination of the beam propagation. However, from other tests and from our general experience, gaussian beam optics are far more reliable quantitatively than this test implies.

3.4 Calculating the Effective Focal Length of Lens

the quoted focal length of the lenses we used was for light at $\lambda = 546.1$ nm. we use $\lambda = 794.7$ nm. the correction is about one percent. the lens focal length is guaranteed to be within one percent. show derivation. propagate this error into data.

3.5 Variation of Gaussian Beam Parameters

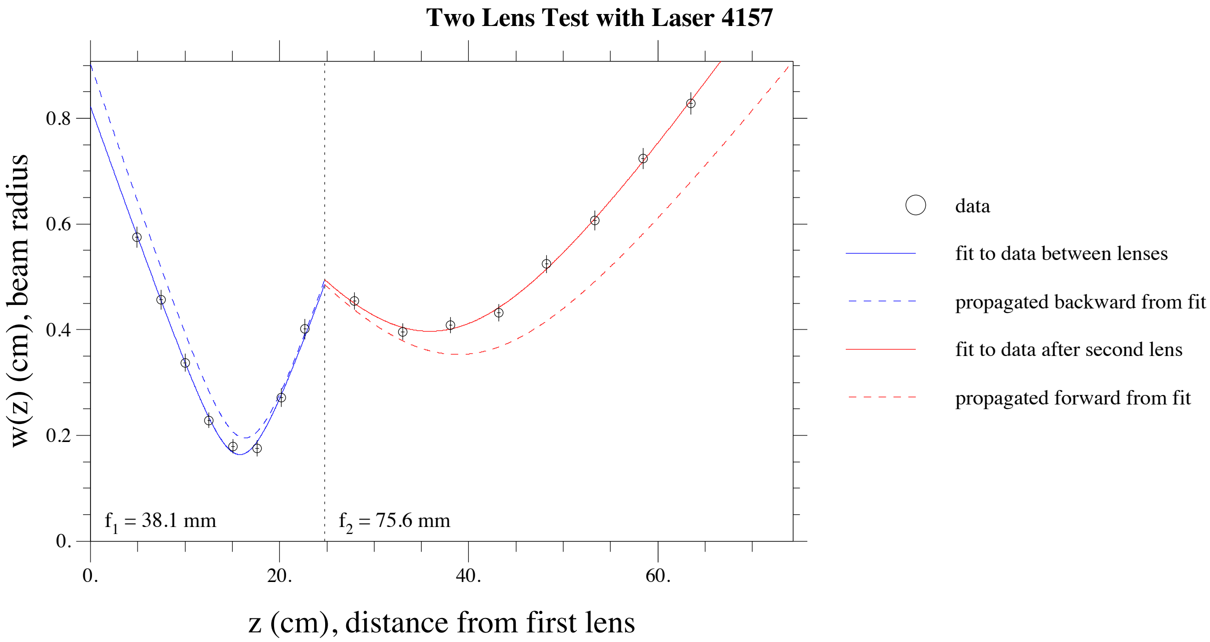
3.6 FAP System with 2.0" Beam Diameter

Me and Al measured:

$$\alpha = 0.008619 \text{ cm} \tag{91}$$
$$w_0 = 0.04126 \text{ cm} \tag{92}$$
$$\psi_0 = 0.2089 \text{ rad} \tag{93}$$
$$z_R = 0.1975 \text{ cm} \tag{94}$$

according to spec sheet:

$$\alpha = 0.008 \text{ cm} \tag{95}$$



fit to data between lenses, χ^2/N_{dof} = 2.11/5

waist radius = 0.1637 ± 0.0099 cm

far field div. = 0.05098 ± 0.00147 cm

waist location = 15.8 ± 0.17 cm

propagated backward from fit

waist radius = 0.19518 ± 0.00663 cm (2.64σ)

far field div. = 0.0539 ± 0.0011 cm (1.59σ)

waist location = 16.343 ± 0.29 cm (1.61σ)

fit to data after second lens, χ^2/N_{dof} = 1.87/5

waist radius = 0.397 ± 0.0075 cm

far field div. = 0.02654 ± 0.00091 cm

waist location = 35.885 ± 0.64 cm

propagated forward from fit

waist radius = 0.353 ± 0.0151 cm (-2.6σ)

far field div. = 0.0236 ± 0.0015 cm (-1.67σ)

waist location = 38.865 ± 1.21 cm (2.17σ)

Figure 2: The first lens ($f_1 = 38.1$ mm) is located at the origin. The second lens ($f_2 = 75.6$ mm), denoted by a dotted line, is located at $z = 24.765 \pm 0.25$ cm. The data on either side of the second lens are fit separately. The solid lines are fits to the data. The fitted gaussian beam before the second lens is propagated forward using the gaussian beam lens transformation equations. The same is done for the fitted gaussian beam after the second lens. The dashed lines represent these propagated curves. The uncertainty bars cover a range of $\pm 1\sigma$. The uncertainty in the power measurement was taken to be a combination of the 0.35% rms intrinsic stability of the laser and the quoted 0.5% full scale power meter display uncertainty. The uncertainty of the distance measurement (z) was taken to be 1.8 mm. These uncertainties were propagated into uncertainties in the beam radius. The uncertainty in the iris radius was taken to be roughly a factor of 3.0 times the average marker resolution of the calipers. Although identical uncertainty analyses were performed for all gaussian beam parameter data sets, the reduced χ^2 for these two data sets are lower than for the other data sets. This is not, strictly speaking, statistically anomalous considering each set had only 5 degrees of freedom. The numbers in paranthesis next to the propagated values are the differences between the propagated value and the fit value scaled to $\sigma = \sqrt{\sigma_p^2 + \sigma_f^2}$. Consistency between the fit and propagated values will be defined to be $< 2\sigma$ by convention. Therefore, the far field divergences are consistent, but the beam waist radii are inconsistent with each other at the 2.6σ level. It is not clear why the fit and propagated values before the second lens appear more consistent. As mentioned earlier, the reduced χ^2 seems very low for both data sets. If this is due to an overestimation of the statistical uncertainties, then, after reducing the uncertainties, every parameter would be inconsistent at the $> 2.3\sigma$ level. On the other hand, had more data been taken (symmetrically distributed about the beam waist), then the fit would have been more constrained. This might have resulted in a greater degree of consistency.

$$w_0 = 0.04 \text{ cm} \quad (96)$$
$$\psi_0 = 0.2 \text{ rad} \quad (97)$$
$$z_R = 0.2 \text{ cm} \quad (98)$$

3.7 FAP System with 3.5" Beam Diameter

Me and Al. (add data)

3.8 FAP System with Single Fiber

James and MC measured:

$$\alpha = 0.008634 \text{ cm} \quad (99)$$
$$w_0 = 0.04778 \text{ cm} \quad (100)$$
$$\psi_0 = 0.1807 \text{ rad} \quad (101)$$
$$z_R = 0.2644 \text{ cm} \quad (102)$$

3.9 FAP System with One Laser in Combiner Fiber

James and MC measured:

$$\alpha = 0.006883 \text{ cm} \quad (103)$$
$$w_0 = 0.03989 \text{ cm} \quad (104)$$
$$\psi_0 = 0.1725 \text{ rad} \quad (105)$$
$$z_R = 0.2312 \text{ cm} \quad (106)$$

3.10 Single Frequency Laser

We should measure the gaussian beam parameters of the of the Ti:Sapphire again. I think that Kai-Mei did this once and I think that Ryan may have also did this once.

4 Combiner

4.1 Description

4.2 Exit Aperture Heating

4.3 Beam Separation Divergence

4.4 Diffuser

5 Choosing the Correct Optics

5.1 Minimum Optics Diameter

How large does an aperture have to be to accomadate a gaussian beam? Unfortunately, the power of a gaussian beam is distributed over an area of infinite size. Therefore any aperture with finite size will clip or cut some of the power of the beam at the edges. In addition, the edge of the beam will be suceptible to diffraction effects. A larger optic or aperture will result in lower power loss and less diffraction at the edges. Choosing an aperture size that allows transmission of 99% of the power is suffiently large enough to minimize power loss and diffraction effects. The minimum optic of aperture diameter d_{min} is:

$$d_{min} = 3w \quad (107)$$

where w is the beam radius at the aperture. Since the beam radius increases as the beam propagates, it is best to keep all smaller apertures as close to the beam waist as possible. Typically only the inner 90% of an optic is considered useful.

5.2 Notes on Lens

The beam waist of a gaussian beam that is exiting an optical fiber is assumed to have the same radius as the output aperture of the fiber. Also, the beam waist is assumed to be located at the exit surface of the optical fiber. A lens can be used to roughly collimate the beam coming out of a fiber. Putting the fiber at the focal length of the lens insures a maximum beam radius (at some point far from the lens) and a minimum far field divergence. Moving the fiber off the focal length in either direction decreases the beam radius (at the same point far from the lens) and increases the far field divergence. The maximum distance that a light source can be placed from a lens depends on the initial far field divergence of the beam coming out of the fiber. The maximum beam radius, w_{max}, that can fit into a lens with a usable diameter, d, is:

$$w_{max} = \frac{d}{3} \tag{108}$$

The maximum beam radius of a beam coming out of a fiber can be estimated by using the far field divergence of the beam:

$$w_{max} = \psi_0 z_{max} \tag{109}$$

Putting these together, the maximum distance that a fiber tip can be placed from a lens with usuable diameter d:

$$z_{max} = \frac{d}{3\psi_0} \tag{110}$$

The far field divergence of the optical fibers that we use is 0.20. Using a 2 in diameter lens with a 90% clear aperture gives a maximum lens to fiber of 3 in = 76.2 mm

REFERENCES!!!! & **Diagrams**