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Today

- Announcements:
  - HW#3 is due tomorrow, Wednesday Jan 30th at 8 am.
  - Look for the next two extra credit problems starting tomorrow
  - HW#4 will be due next Wednesday, Feb 6th
- Vectors
- Special Relativity

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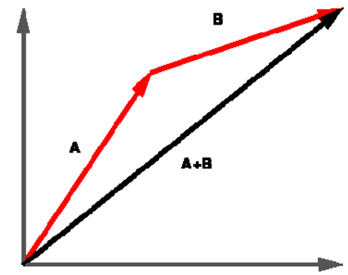
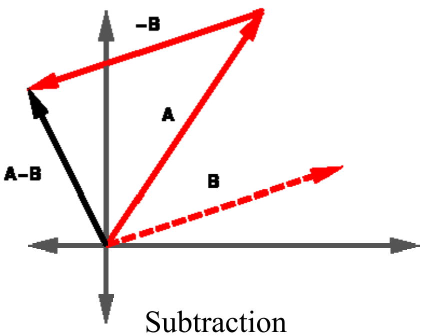
Vector Addition and Subtraction



Addition



Subtraction

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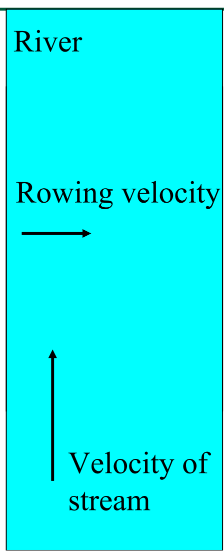
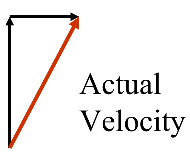
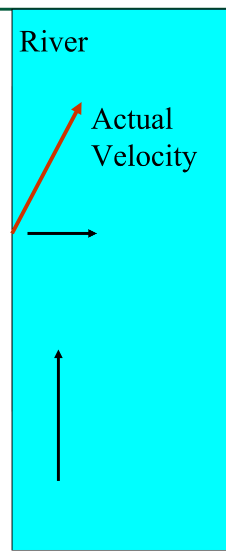
Example of the use of vectors







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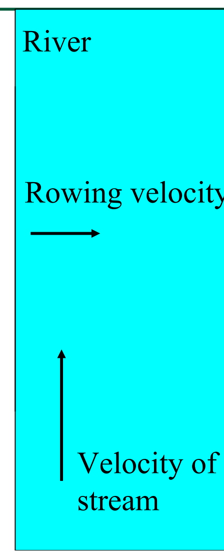
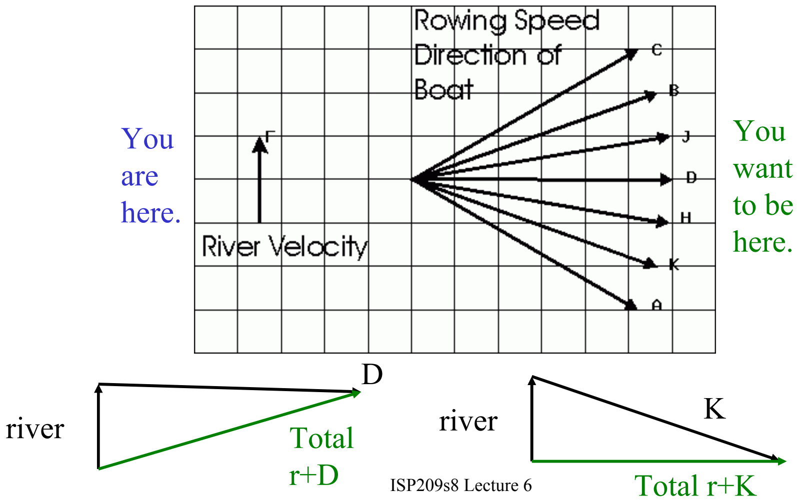
How long to cross the river?



$$\text{time} = \frac{\text{distance across the river}}{\text{rowing speed ACROSS}}$$

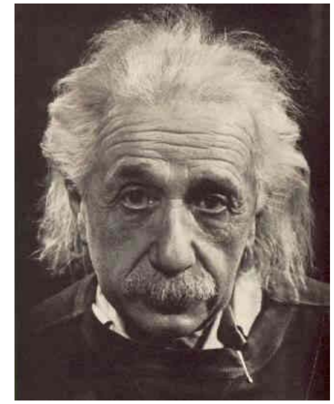
The rowing speed ACROSS is the magnitude of the boat’s velocity directed toward the opposite bank.

Vectors



Albert Einstein

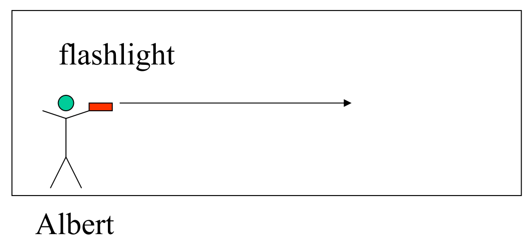
- Poor high school student – did not talk till age 5
- Patent clerk in 1905
- In 1905 published three papers that changed scientific thought
  - Special relativity (moving clocks run slow)
  - Quantization of light (foundation of quantum mechanics)
  - Brownian motion (foundation of statistical physics)
- Was able to do this, in part, by asking simple questions
- Train story…



The principle of relativity

Every nonaccelerated observer observes the same laws of nature. In other words, no experimental performed within a sealed room moving at an unchanging speed and direction can tell you whether you are standing still or moving.

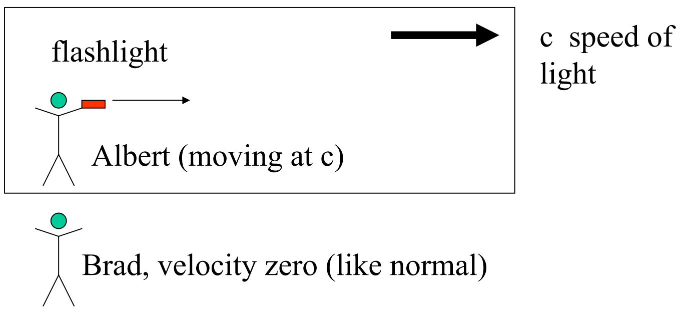
A consequence – speed of light



What is the speed Albert measures for light?

A). 3.0E8 m/s (we often write this as c)

A consequence – speed of light



What is the speed Brad measures for the light of Albert's flashlight?

A). 3.0E8 m/s (we often write this as c)

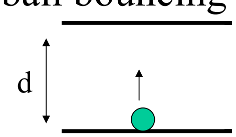
B). c + c = 6.0E8 m/s

Einstein's Postulates of Special Relativity

- The laws of physics must be the same in all inertial (nonaccelerating) reference frames.
- The speed of light in empty space is a constant in all inertial frames, regardless of the motion of the source or the observer.

A simple clock

- A perfectly elastic ball bouncing between two fixed walls:



- One click:

$$\text{time} = \frac{\text{distance}}{\text{speed}} \quad or \quad \text{distance} = \text{speed} \cdot \text{time}$$

$$\text{time for a click} = \frac{2 \times \text{d}}{\text{speed}}$$

$$click = \frac{2 \times 1m}{2m/s} = 1s$$

Consequences of Special Relativity

- Clocks in moving systems run more slowly.
  - Equations: $\beta = v/c$ $\quad t = \gamma\, t_0 \quad \gamma = \frac{1}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}} = \frac{1}{\sqrt{1-\beta^2}}$
  - t_0 is called the "proper" time it is the time measure by the clock you can hold in your hand.
  - c = speed of light = 299 792 458 m / s
- The length of moving objects is smaller
  - l_0 is the "proper" length $\quad l = \frac{l_0}{\gamma} \quad \gamma = \frac{1}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}} = \frac{1}{\sqrt{1-\beta^2}}$
- How do we know?
  - Clock in airplanes, clock at the North Pole compared to the equator
  - Lifetime of fundamental particles

Example

Two identical clocks are used in an experiment. One is kept on Earth and the other passes by in a space ship traveling at 0.5c. If according to us on Earth the time between two events is 2.00 s, what is the time interval according to the clock on the ship?

$$t = \sqrt{\frac{1}{1-\left(\frac{v^2}{c^2}\right)}}\, t_0 \quad = \sqrt{\frac{1}{1-\left(\frac{(0.5c)^2}{c^2}\right)}}\, t_0 = \sqrt{\frac{1}{1-\left((0.5)^2\right)}}(2.00s) = 2.31s$$

We can hold the clock.

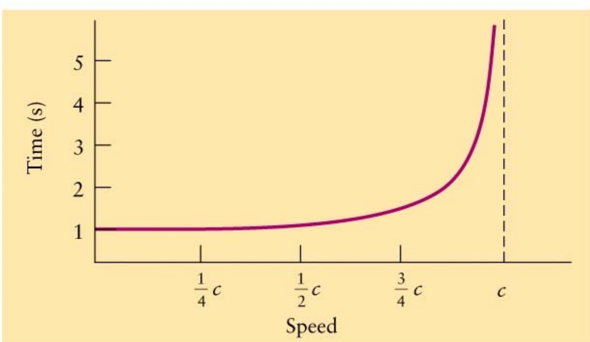
Length Contraction

If we fly to the nearest star (4 light years away) at a speed of 0.9c. How long would we think the distance is?

$$l = \frac{l_0}{\sqrt{\frac{1}{1-\left(\frac{v^2}{c^2}\right)}}} = \frac{4ly}{\sqrt{\frac{1}{1-\left(\frac{(0.9c)^2}{c^2}\right)}}} = \frac{4ly}{\sqrt{\frac{1}{1-0.81}}} = 1.74ly$$

The length in the frame where the star and Earth are; proper length.

Time Dilation



v/c	γ
.1	1.00504
.2	1.02062
.3	1.04828
.4	1.09109
.5	1.1547
.6	1.25
.7	1.40028
.8	1.6667
.9	2.29416
1	∞

World record v/c (for electrons) is from SLAC in California: 0.999999875

$\gamma = 20{,}000$

The Twin Paradox

- Upon birth one twin is put in a space ship and flown at the speed of light for 20 Earth years. The other twin stays on Earth.
- After 20 years the ship returns.
- The twin on Earth is 20 years old and the twin in the ship is still a new-born, less than a day old.
- But for the twin on the ship it looked like the twin on Earth was moving. They expect upon return to be older than twin on Earth.
- This is called the twin paradox.
- Actually there is no paradox because the twin in motion had to accelerate

Another consequence of special relativity

- F = ma.
- What happens as we accelerate to near the speed of light? We can't continue to accelerate. If we did we would exceed the speed of light and that is not allowed.
- Newton's law must be valid, so it must be that mass increases near the speed of light. Mass is relative!
- We define the rest mass as the mass of an object at rest.

This lead Einstein to his famous equation…

- Energy is the ability to do work. It comes in two main types
  - Kinetic energy: the energy of motion
  - Potential energy: the energy of position
- Work = force x distance (it's a scalar measured in Joules, J (same as Nm))
- Einstein's Energy-mass relation:

$$E = m\,c^2$$

Picture

<u>The following is a picture of a chemical reaction:</u>

Start with some initial mass (kg) → Something happens → End up with some final mass (kg)

Some fraction, f, is converted to energy ↓ f- see the table on the next page

ENERGY (Joules, J)

The amount of energy is $E = m_{converted}\,c^2$

$m_{converted}$ = (Mass to start) x fraction

Fraction of Energy Converted

- In a chemical reaction not all the mass can be converted to energy. Actually only a very small fraction (the exact value of the fraction depends on the chemical reaction).

Reaction	Fraction	Example
Matter-Antimatter Annihilation	1	No common example; happens at particle accelerators
Fusion	0.007	Power source of the Sun
Fission	0.001	Nuclear power plant
Chemical	1×10^{-10}	Burning coal
Mechanical	1×10^{-15}	Compressing a spring

Clicker Question

What process requires the least mass to produce energy?

A) matter-antimater annihilation

B) Fusion in the Sun

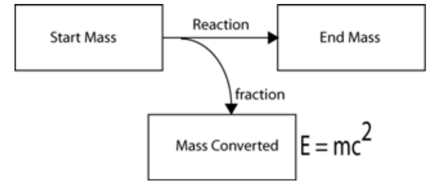
C) Fission at the Fermi II nuclear power plant

D) Coal generation

E) Wind power

HW Help: How long will the Sun burn?

The sun generates 3.82E24 W of power by fusion of hydrogen into helium. The fraction of mass converted for fusion is 0.007. How many kg of protons and electrons does the Sun use every second?



$$m_{burned\ each\ s} = \frac{m_{converted}}{f} = \frac{E/c^2}{f} = \frac{3.82E24\ J \Big/ \left(3E8\ m/s\right)^2}{0.007} = 6.06E9\ kg$$

Years Sun will last = (Total mass of the core/mass used per second) x (years/s)

Note: 1 year = 3.156E+7 s