Chapter 12: Non-Perturbative Approaches For Many Fermion Systems

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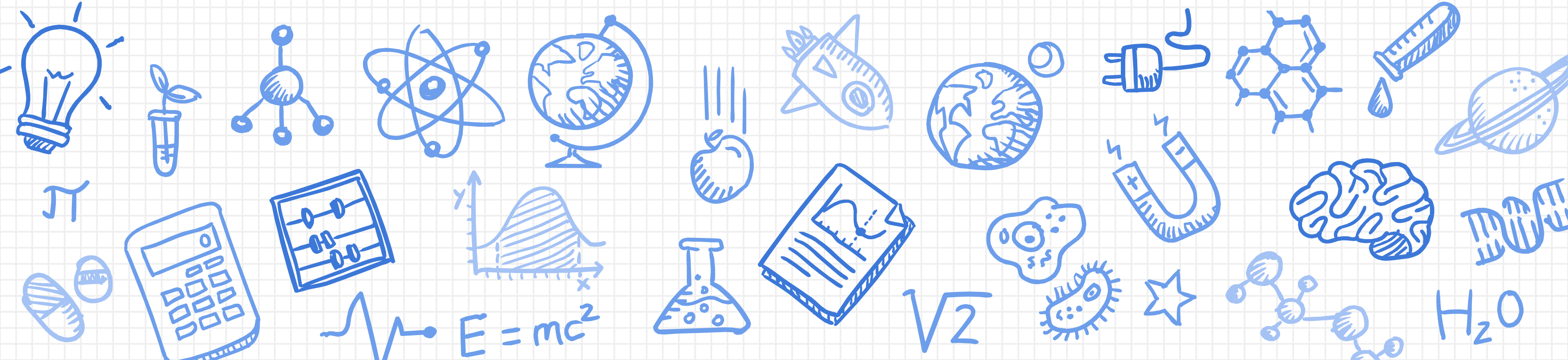

Important Topics

- The Thomas-Fermi Approximation
- The Periodic Table
- Configuration Splitting and Hund's Rules
- Constructing States According to Total L and Total S
- Zeeman Shift and the Integer Quantum Hall Effect

The Thomas–Fermi Approximation

- Given the potential and chemical potential you can find the density
- Valid for large N and slowly changing potentials

$$\frac{\hbar^2 k_f(\vec{r})^2}{2m} + V(\vec{r}) = \mu_f$$

$$n = \frac{(2s+1)}{(2\pi)^d} \int_{k<k_f} d^d k, \; d \text{ dimensions}$$

The Periodic Table

- Having multiple electrons leads to screening of the positive charge of the nucleus
- States that have higher probability of being closer to the nucleus have lower energy
- Radial wave functions behave as R^L near the origin
- For a given N the state with lower L will have lower energy
- The set of orbitals with a given N and L are known as a shell
- The degeneracy of a shell is (4*L + 2)

Configuration Splitting and Hund's Rules

To ID lowest Energy State

1. State with the largest S
2. For a given S, LS multiplet with the largest L.
3. For states with shells less than half filled, the level with the smallest J have (behave the opposite way for shells that are more than half filled).

$$^{(2S+1)}L_J$$

Permutation Symmetry

- Overall wave function must be antisymmetric
- For 2 particles in same shell ($l_1=l_2$)
    - Spatial WF (L): Symmetry goes as $(-1)^L$
    - Spin WF (S): Symmetric (Antisymmetric) for S=1 (0)
- Mixed symmetries for 3+ particles

Magnetism Stuff

- Zeeman Splitting:

$$\Delta E = -g\frac{e\hbar B}{2mc}M_J$$

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

- Quantized Hall Effect
  - n = conduction electrons/unit area
  - m = natural number
  - Lowest m gives highest B for

$$B = \frac{2\pi\hbar cn}{em}$$

Possible Problem

- ✘ Chapter 12, Problem 1
- ✘ Chapter 12, Problem 4

Chapter 12, Problem 1

Consider the two electron holes in the p-shell of a neutral oxygen atom.

A. What is the $L - S - J$ of the ground state

B. If the atom is in a magnetic field of 0.01 Tesla, find the magnetic energies of the originally degenerate $2J + 1$ states.

Chapter 12, Problem 4

Consider a surface with 10 electrons per μm^2 . Lowering the magnetic field, at what magnetic field (in Tesla) do you find the first dip in conductivity due to the quantum Hall effect?