

your name(s) _____

Physics 852 Exercise #8 - Friday, March. 18th

BCS theory is built on a simplified assumption of an interaction with a fixed density of states over a finite energy range, with an interaction that mixes all states (with a given total momentum) equally. To mimic this, consider the following Hamiltonian in matrix form,

$$\begin{aligned} H_{ii} &= E_0(i/M), & E_0 &= 1.0 \\ H_{i \neq j} &= -g/M, & g &= 0.1; \end{aligned} \quad (0.1)$$

where M is the dimensionality of the matrix with $0 \leq i < M$.

1. Write a program that finds the eigenvalues of the matrix and prints out the lowest 10 eigenvalues.
2. Run the program for $M = 5, 20, 80, 320$ and compare results.
3. For $M = 320$, compare the lowest eigenvalues for $g = 0.1, 0.01, 0.001$.
4. What happens under the interchange $g \rightarrow -g$?

Solution:

For $g = 0.1$

5	20	80	320
-0.00476257	-0.00249910	-0.00110069	-0.00046784
0.19876418	0.04861970	0.01181788	0.00283391
0.40048017	0.09904822	0.02446164	0.00601242
0.60190027	0.14931198	0.03704511	0.00916734
0.80361795	0.19950424	0.04960292	0.01231256
0.00000000	0.24965758	0.06214675	0.01545262
0.00000000	0.29978706	0.07468189	0.01858949
0.00000000	0.34990091	0.08721115	0.02172423
0.00000000	0.4000420	0.09973618	0.02485745
0.00000000	0.45010034	0.11225803	0.02798954

For $g = 0.01$

5	20	80	320
-0.00004225	-0.00001830	-0.00000649	-0.00000211
0.19998368	0.04998734	0.01249491	0.00312325
0.40000050	0.09999026	0.02499560	0.00624842
0.60001699	0.14999230	0.03749607	0.00937354
0.80004109	0.19999390	0.04999642	0.01249862
0.00000000	0.24999527	0.06249670	0.01562369
0.00000000	0.29999648	0.07499694	0.01874875
0.00000000	0.34999759	0.08749714	0.02187380
0.00000000	0.39999864	0.09999732	0.02499884
0.00000000	0.44999965	0.11249749	0.02812388

For $g = 0.001$

5	20	80	320
-0.00000042	-0.00000018	-0.00000006	-0.00000002
0.19999983	0.04999988	0.01249995	0.00312498
0.40000000	0.09999990	0.02499996	0.00624998
0.60000017	0.14999992	0.03749996	0.00937499
0.80000042	0.19999994	0.04999996	0.01249999
0.00000000	0.24999995	0.06249997	0.01562499
0.00000000	0.29999996	0.07499997	0.01874999
0.00000000	0.34999997	0.08749997	0.02187499
0.00000000	0.39999999	0.09999997	0.02499999
0.00000000	0.45000000	0.11249997	0.02812499

For $g = -0.01$

5	20	80	320
-0.00361795	-0.00134267	-0.00042037	-0.00012251
0.19809973	0.04893659	0.01213474	0.00301395
0.39951983	0.09910699	0.02466642	0.00614540
0.60123582	0.14923778	0.03718934	0.00927499
0.80476257	0.19934867	0.04970762	0.01240360
0.00000000	0.24944828	0.06222301	0.01553159
0.00000000	0.29954133	0.07473642	0.01865916
0.00000000	0.34963087	0.08724839	0.02178642
0.00000000	0.39971916	0.09975926	0.02491345
0.00000000	0.44980813	0.11226928	0.02804028