$\qquad$

Physics 852 Exercise 5 - Friday, Feb. 18th
Consider a harmonic oscillator, which you will fill up with neutrons. The single-particle levels are originally separated by energies $\hbar \omega$. A spin-orbit interaction is then added,

$$
H_{\text {s.o. }}=-\frac{\beta}{\hbar^{2}} \vec{L} \cdot \vec{S}=-\beta[j(j+1)-\ell(\ell+1)-3 / 4] / 2,
$$

and a final term is added that depends only on $\ell$,

$$
H_{L}=-\frac{\alpha}{\hbar^{2}}|\vec{L}|^{2}=-\alpha[\ell(\ell+1)] .
$$

Choose $\beta=0.12 \hbar \omega$ and $\alpha=0.02 \hbar \omega$. Let $N$ refer to the original quantum excitation in the absence of the the spin-orbit and $\ell$-dependent interactions, i.e. $E=\hbar \omega(N+3 / 2)$.
Make an energy-level diagram for all levels with $\boldsymbol{N} \leq \mathbf{7}$. Plot the energies $\boldsymbol{E}_{\boldsymbol{i}}$ in units of $\hbar \boldsymbol{\omega}$ (set $\hbar \boldsymbol{\omega}=\mathbf{1}$ ). Label each level by $\boldsymbol{N}, \boldsymbol{j}, \boldsymbol{\ell}$ and $\boldsymbol{E}_{\boldsymbol{i}}$. The level diagram should consist of a short horizontal line for each level, where the height is proportional to $\boldsymbol{E}_{\boldsymbol{i}}$.
If you create data files in the form:

| \#Asum | E | delE | N | ell | $j$ |
| ---: | :---: | :--- | :--- | :---: | :---: |
| 2 | 0.000 | 0.000 | 0 | 0 | 0.5 |
| 6 | 0.900 | 0.900 | 1 | 1 | 1.5 |
| 8 | 1.080 | 0.180 | 1 | 1 | 0.5 |
| 14 | 1.760 | 0.680 | 2 | 2 | 2.5 |
| 16 | 2.000 | 0.240 | 2 | 0 | 0.5 |
| 20 | 2.060 | 0.060 | 2 | 2 | 1.5 |
| 28 | 2.580 | 0.520 | 3 | 3 | 3.5 |
| 32 | 2.900 | 0.320 | 3 | 1 | 1.5 |

You can use the python program emailed to you to make the plot. The second column is the energy of a single-particle level and the fourth column is the label $\boldsymbol{N}$, i.e. in absence of last two terms in Hamiltonian, $\boldsymbol{E}=\boldsymbol{N} \hbar \boldsymbol{\omega}$. The third column is the difference in adjacent energy levels, and the first column is the sum of all neutron number of all the levels up to and including the given level.
It is not required, but if you use maps (in $\mathrm{C}++$ ) or dictionaries (in python) you can have the list of energy levels ordered for you. Otherwise, you may need to rearrange the output by hand.

## Solution:



