your name(s)____

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Physics 852 Exercise #3 - Friday, Feb. 5th

The ω meson (mass=782 MeV) is charge neutral and has total isospin $I = 0$. For reasons we won't explain (g-parity) it mainly decays to a 3-pion channel. The pions, π^+, π^0, π^- are an I=1 isotriplet. If you couple the three isospins together, the projections of the pion's isospin, m_1, m_2, m_3, couple to total isospin I and I_{12} and projection M. You must first couple the first two spins, m_1, m_2 to I_{12} and spin M_{12}. Then you couple I_{12}, M_{12} to m_3 to get the I, M states.

In other words:

1. Write the $|I_{\rm total} = 0, M = 0\rangle$ state in terms of $|I_{12}, M_{12}, m_3\rangle$ states.
2. Write the $|I_{12}, M_{12}, m_3\rangle$ states in terms of the $|m_1, m_2, m_3\rangle$ states, where m_3 goes along for the ride.
3. Express the $|I_{\rm total} = 0, M = 0\rangle$ state in terms of a sum over combinations $|m_1, m_2, m_3\rangle$ states.

PART I.

1. What values of I_{12} contribute to the ω decay?
2. Write the isospin portion of the ω wave function for pions with final momenta $\vec{k}_1, \vec{k}_2, \vec{k}_3$ as a sum of products of terms of the form, e.g. $\pi_1^+ \pi_2^- \pi_3^0$.
$$|\omega\rangle = \boxed{?}|\pi^+\pi^-\pi^0\rangle + \boxed{?}|\pi^-\pi^+\pi^0\rangle + \boxed{?}|\pi^0\pi^-\pi^+\rangle + \cdots ,$$
where you need to fill in the boxes. In the first term the pion with momentum $\vec{k}_1$ is positive, the pion with momentum $\vec{k}_2$ is negative and the pion with momentum $\vec{k}_3$ is neutral.
3. What are the branching ratios to various combinations of m_1, m_2, m_3 for the ω decay? I.e. what fraction of the decays are $\pi^0\pi^0\pi^0$ and what fraction are $\pi^+\pi^-\pi^0$?

PART II. (EXTRA CREDIT)

You can re-write the pion states in a "Cartesian" basis as π_x, π_y, π_z defined by
$$\begin{aligned} \pi_z &= \pi_0 \\ \pi_x &= (\pi^- - \pi^+)/\sqrt{2}, \\ \pi_y &= i(\pi^+ + \pi^-)/\sqrt{2}. \end{aligned}$$

1. Using your Clebsch-Gordan skills write the scalar combination
$$S = \vec{\pi}_1 \cdot \vec{\pi}_2 = \pi_{1,x}\pi_{2,x} + \pi_{1,y}\pi_{2,y} + \pi_{1,z}\pi_{2,z}$$
in terms of $\pi_i^{+,0,-}$ operators.
2. In the Cartesian basis, using $\vec{\pi}_1, \vec{\pi}_2, \vec{\pi}_3$, write an expression involving all three labels (1,2,3) with pion fields to the 3$^{\rm rd}$ order that is an isoscalar. (You may want to use cross products)
3. Rewrite this in terms of the $\pi^{+,0,-}$ basis.

Potentially Useful Information: You can use the following for coupling two multiplets with $j_1 = j_2 = 1$. From HW:

$$|J=0, M=0\rangle = (|m_1=1, m_2=-1\rangle + |m_1=-1, m_2=1\rangle - |m_1=0, m_2=0\rangle)/\sqrt{3}.$$

From lecture notes:

$$|J=1, M=1\rangle = (|m_1=1, m_2=0\rangle - |m_1=0, m_2=1\rangle)/\sqrt{2},$$
$$|J=1, M=0\rangle = (|m_1=1, m_2=-1\rangle - |m_1=-1, m_2=1\rangle)/\sqrt{2},$$
$$|J=1, M=-1\rangle = (|m_1=0, m_2=-1\rangle - |m_1=-1, m_2=0\rangle)/\sqrt{2}.$$

Solutions:

1. only $I_{12} = 1$ can couple with $I_3 = 1$ to give $I = 0$.
2. Using the first equation above, with $m_1 \to M_{12}$ and $m_2 \to m_3$ and $J \to I$,

   $$|I=0, M=0\rangle = \frac{1}{\sqrt{3}}\{|M_{12}=1, m_3=-1\rangle + |M_{12}=-1, m_3=1\rangle - |M_{12}=0, m_3=0\rangle\}$$

   Now plugging in the lower expressions where $M \to M_{12}$,

   $$|I=0, M=0\rangle = \frac{1}{\sqrt{6}}\{|+,0,-\rangle - |+,-,0\rangle - |0,+,-\rangle + |0,-,+\rangle + |-,+,0\rangle - |-,0,+\rangle\}$$

3. There is no strength to $|0,0,0\rangle$, so the decay goes 100% to $\pi^+\pi^-\pi^0$.

Solutions to Extra Credit:

1.

$$S = \pi_1^0\pi_2^0 + \pi_1^+\pi_2^- + \pi_1^-\pi_2^+.$$

2.

$$(\vec{\pi}_1 \times \vec{\pi}_2) \cdot \vec{\pi}_3.$$

3.

$$\begin{aligned}
(\vec{\pi}_1 \times \vec{\pi}_2) \cdot \vec{\pi}_3 = {} & \pi_{1x}\pi_{2y}\pi_{3z} - \pi_{1y}\pi_{2x}\pi_{3z} + \pi_{1y}\pi_{2z}\pi_{3x} - \pi_{1z}\pi_{2y}\pi_{3x} + \pi_{1z}\pi_{2z}\pi_{3x} - \pi_{1x}\pi_{2z}\pi_{3y} \\
& \frac{1}{2}\Big[i(\pi_1^- - \pi_1^+)(\pi_2^+ + \pi_2^-) - i(\pi_1^+ + \pi_1^-)(\pi_2^- - \pi_2^+)\pi_3^0 \\
& + i(\pi_2^- - \pi_2^+)(\pi_3^+ + \pi_3^-) - i(\pi_2^+ + \pi_2^-)(\pi_3^- - \pi_3^+)\pi_1^0 \\
& + i(\pi_3^- - \pi_3^+)(\pi_1^+ + \pi_1^-) - i(\pi_3^+ + \pi_3^-)(\pi_1^- - \pi_1^+)\pi_2^0\Big] \\
& = -i\pi_1^+\pi_2^-\pi_3^0 + i\pi_1^-\pi_2^+\pi_3^0 + i\pi_1^+\pi_2^0\pi_3^- - i\pi_1^-\pi_2^0\pi_3^+ - i\pi_1^0\pi_2^+\pi_3^- + i\pi_1^0\pi_2^-\pi_3^+.
\end{aligned}$$

Aside from overall constant in $(i/\sqrt{6})$, this is the same as answer to number 2.