your name(s)____

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Physics 852 Exercise #3 - Friday, Jan. 28th

The ω meson (mass=782 MeV) is charge neutral and has total isospin $I = 0$. For reasons we won't explain (g-parity) it mainly decays to a 3-pion channel. The pions, π^+, π^0, π^- are an I=1 isotriplet. If you couple the three isospins together, the projections of the pion's isospin, m_1, m_2, m_3, couple to total isospin I and I_{12} and projection M. You must first couple the first two spins, m_1, m_2 to I_{12} and spin M_{12}. Then you couple I_{12}, M_{12} to m_3 to get the I, M states.

In other words:

1. Write the $|I_{\rm total} = 0, M = 0\rangle$ state in terms of $|I_{12}, M_{12}, m_3\rangle$ states.
2. Write the $|I_{12}, M_{12}, m_3\rangle$ states in terms of the $|m_1, m_2, m_3\rangle$ states, where m_3 goes along for the ride.
3. Express the $|I_{\rm total} = 0, M = 0\rangle$ state in terms of a sum over combinations $|m_1, m_2, m_3\rangle$ states.

PART I.

1. What values of I_{12} contribute to the ω decay?
2. Write the isospin portion of the ω wave function for pions with final momenta $\vec{k}_1, \vec{k}_2, \vec{k}_3$ as a sum of products of terms of the form, e.g. $\pi_1^+ \pi_2^- \pi_3^0$.
$$|\omega\rangle = \boxed{?}|\pi^+\pi^-\pi^0\rangle + \boxed{?}|\pi^-\pi^+\pi^0\rangle + \boxed{?}|\pi^0\pi^-\pi^+\rangle + \cdots,$$
where you need to fill in the boxes. In the first term the pion with momentum $\vec{k}_1$ is positive, the pion with momentum $\vec{k}_2$ is negative and the pion with momentum $\vec{k}_3$ is neutral.
3. What are the branching ratios to various combinations of m_1, m_2, m_3 for the ω decay? I.e. what fraction of the decays are $\pi^0\pi^0\pi^0$ and what fraction are $\pi^+\pi^-\pi^0$?

PART II. (EXTRA CREDIT)

You can re-write the pion states in a "Cartesian" basis as π_x, π_y, π_z defined by

$$\begin{aligned} \pi_z &= \pi_0 \\ \pi_x &= (\pi^- - \pi^+)/\sqrt{2}, \\ \pi_y &= i(\pi^+ + \pi^-)/\sqrt{2}. \end{aligned}$$

1. Using your Clebsch-Gordan skills write the scalar combination
$$S = \vec{\pi}_1 \cdot \vec{\pi}_2 = \pi_{1,x}\pi_{2,x} + \pi_{1,y}\pi_{2,y} + \pi_{1,z}\pi_{2,z}$$
in terms of $\pi_i^{+,0,-}$ operators.
2. In the Cartesian basis, using $\vec{\pi}_1, \vec{\pi}_2, \vec{\pi}_3$, write an expression involving all three labels (1,2,3) with pion fields to the 3$^{\rm rd}$ order that is an isoscalar. (You may want to use cross products)
3. Rewrite this in terms of the $\pi^{+,0,-}$ basis.

Potentially Useful Information: You can use the following for coupling two multiplets with $j_1 = j_2 = 1$. From HW:

$$|J = 0, M = 0\rangle = (|m_1 = 1, m_2 = -1\rangle + |m_1 = -1, m_2 = 1\rangle - |m_1 = 0, m_2 = 0\rangle)/\sqrt{3}.$$

From lecture notes:

$$\begin{aligned}
|J = 1, M = 1\rangle &= (|m_1 = 1, m_2 = 0\rangle - |m_1 = 0, m_2 = 1\rangle)/\sqrt{2}, \\
|J = 1, M = 0\rangle &= (|m_1 = 1, m_2 = -1\rangle - |m_1 = -1, m_2 = 1\rangle)/\sqrt{2}, \\
|J = 1, M = -1\rangle &= (|m_1 = 0, m_2 = -1\rangle - |m_1 = -1, m_2 = 0\rangle)/\sqrt{2}.
\end{aligned}$$