your name(s)____

____

Physics 852 Exercise #1 - Friday, Jan. 16nd

Consider two kinds of spinless particles, whose masses are m_A and m_B. The particles exist in a one-dimensional world. We define field operators,

$$\Phi_A(x) = \sum_k \frac{1}{\sqrt{LE_A(k)}} \left(a_k e^{ikx} + a_k^\dagger e^{-ikx} \right),$$
$$\Phi_B(x) = \sum_k \frac{1}{\sqrt{LE_B(k)}} \left(b_k e^{ikx} + b_k^\dagger e^{-ikx} \right).$$

Here, L is some large length. The interaction Hamiltonian is

$$H_{\rm int} = g \int dx\ \Phi_A(x)\Phi_B(x)^2. \qquad (0.1)$$

Now, let $m_A > 2m_B$, so that the heavier A particle can decay into two lighter B particles. Also assume the decay energy is sufficiently high that the lighter particles move relativistically, $E_B(k)^2 = (\hbar c)^2 k^2 + m_B^2$.

1. Calculate the matrix element $\mathcal{M} = \langle k_{B1}, k_{B2} | H_{\rm int} | k_A = 0 \rangle$. Use the orthogonality of the momentum states:
$$\int dx\ e^{ik_1 x} e^{ik_2 x} = L\delta_{k_1,-k_2}.$$
Your answer should contain a Kronecker delta.
2. Calculate the decay rate, Γ, for the reaction $A \to 2B$ in lowest order perturbation theory. Express your answer in terms of m_A, m_B and g.
3. We have been working in units where m_A and m_B have units of energy. What are the dimensions of g? Check the dimensional consistency of your answer for Γ.