your name(s)____

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Physics 851 Exercise #5

Consider two nucleons, mass = 939 MeV/c^2. They bind into a deuteron with a binding energy of 2.2 MeV. Consider a potential,

$$V(r) = \begin{cases} \infty, & r < 0 \\ -V_0/[1 + e^{(r-a)/a}], & r > 0 \end{cases},$$

where $a = 0.707$ fm. Perform the following calculations numerically.

1. Find V_0 so that the binding energy is indeed $B = 2.2$ MeV. You can treat this as a one-dimensional problem, where $r < 0$ is suppressed by an infinite repulsive potential. Also, don't forget to use the reduced mass $\mu = M/2$. Schrödinger's equation for an s-wave is the same as for a one-dimensional problem,

$$-\frac{\hbar^2}{2\mu}\partial_r^2\phi_0(r) + V(r)\phi_0(r) = E\phi_0(r).$$

   The boundary condition is that $\phi_0(r = 0) = 0$. You can assume that for large r the wave function behaves as e^{-qr}, with q chosen according to the binding energy. Integrate numerically to $r = 0$, then repeat with different values of V_0 until you get $\phi(0) = 0$.

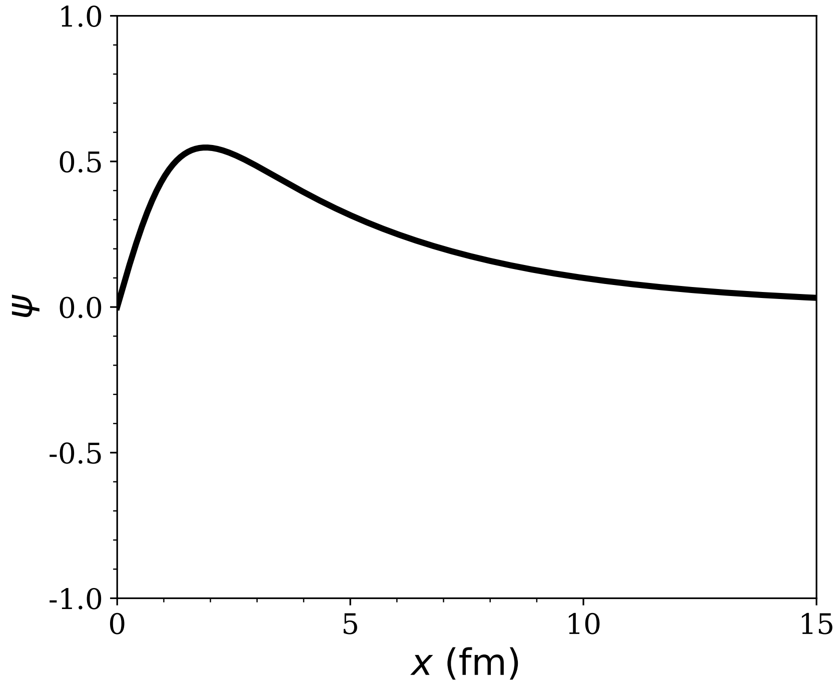
2. What is r.m.s. radius?

$$R^2 = \frac{\int dr\, r^2|\phi_0(r)|^2}{\int dr\, |\phi_0(r)|^2}.$$

FYI: $\hbar c = 197.327$ MeV fm. If masses are int units of MeV/c^2, energies are in MeV, momenta are in units of MeV/c, and distances are in units of fm, Schrödinger's equation becomes

$$-\frac{(\hbar c)^2}{2\mu c^2}\partial_r^2\phi_0(r) + V(r)\phi_0(r) = E\phi_0(r),$$
$$-\partial_r^2\phi_0(r) = (-q^2 - 2\mu c^2 V(r)/(\hbar c)^2)\phi_0(r).$$

and $\mu c^2 = 469.5$ MeV, while $q^2 = 2\mu c^2 B/(\hbar c)^2$. In these units $\hbar$ always appear as the combination $\hbar c$, and mc^2 is in units of MeV. One can then ignore the factors of c and treat energies, masses and momenta as if they are all in the same units.



PYTHON CODE

```
import matplotlib.pyplot as plt
from matplotlib.lines import Line2D
import numpy as np
import math
import os
from pylab import *
sformatter=ScalarFormatter(useOffset=True,useMathText=True)
sformatter.set_scientific(True)
sformatter.set_powerlimits((-2,3))

NMAX=10000
WF=zeros(NMAX+1)
XMAX=40.0
DX=XMAX/NMAX
a=0.707
E=-2.2
hbarc=197.327
mu=0.5*939.0
x=DX*arange(0,NMAX+1)

def GetQ2(x,V0):
  V=-V0/(1.0+exp((x-a)/a))
  q2=(2.0*mu*(-E+V))/(hbarc*hbarc)
  return q2

def GetIntercept(V0):
  print('here goes')
```

```
  q=sqrt(-2.0*mu*E)/hbarc
  WF[NMAX]=exp(-q*NMAX*DX)
  WF[NMAX-1]=exp(-q*(NMAX-1)*DX)
  N=NMAX-1
  while N>0:
    q21=GetQ2(N*DX,V0)
    WF[N-1]=(2.0*WF[N]-WF[N+1])+q21*DX*DX*WF[N]
    N=N-1

  return WF[0]

V0 = float(input("Enter V0: "))
intercept=GetIntercept(V0)
print('Intercept=',intercept)

plt.rc('text', usetex=False)
plt.figure(figsize=(6,5))
fig = plt.figure(1)
ax = fig.add_axes([0.15,0.12,0.8,0.8])

plt.plot(x,WF,linewidth=3,color='k',label='$\\beta a$=0.5')

ax.tick_params(axis='both', which='major', labelsize=14)
ax.set_xticks(np.arange(0,20,5),minor=False)
ax.set_xticklabels(np.arange(0,20,5),minor=False, family='serif')
ax.set_xticks(np.arange(0,10,1),minor=True)
plt.xlim(0.0,15)
ax.set_yticks(np.arange(-10,10,0.5), minor=False)
ax.set_yticklabels(np.arange(-10,10,0.5), minor=False, family='serif')
ax.set_yticks(np.arange(-10,10,0.1), minor=True)
plt.ylim(-1.0,1.0)

plt.xlabel('$x$ (fm)', fontsize=18, weight='normal')
plt.ylabel('$\psi$',fontsize=18)

plt.savefig('exercise5.pdf',format='pdf')
os.system('open -a Preview exercise5.pdf')
#plt.show()
quit()
```