your name(s)_____

_____

Physics 851 Exercise #4

Consider a particle of mass m under the influence of the potential,

$$V(x) = V_0\theta(-x) - \frac{\hbar^2}{2m}\beta\delta(x-a), \quad V_0 \to \infty,\ \beta > 0.$$

A plane wave moving in the $-\hat{x}$ direction is reflected off the potential. For($x > a$) the plane wave will have the form

$$e^{-ikx} - e^{2i\delta}e^{ikx}.$$

Here, δ is referred to as the phase shift.

1. Find δ as a function of ka, and plot for $\beta a = 0.5$ and for $0 < ka < 10$. Because addition of $n\pi$ to the phase shift is arbitrary, translate all phase shifts to angles between zero and π.

Solution:
B.C.: Choose wave function form

$$\psi_1 = A\sin(kx), \ \ \psi_2 = \sin(kr + \delta).$$

B.C.:

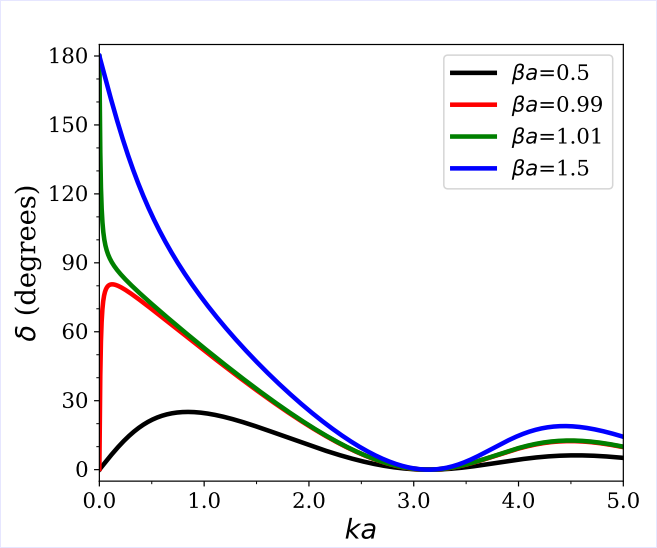
$$A\sin(ka) = \sin(ka + \delta), \ -kA\cos(ka) + \beta A\sin(ka) \qquad = k\cos(ka + \delta).$$

Divide the B.C. to eliminate A,

$$\frac{\sin(ka)}{k\cos(ka) - \beta\sin(ka)} = \frac{1}{k}\tan(ka + \delta), \ \delta \quad = -ka + \tan^{-1}\left(\frac{\sin(ka)}{\cos(ka) - (\beta/k)\sin(ka)}\right)$$

2. Repeat for $\beta a = 0.99, 1.01, 1.5\beta$.

Solution:
Plots for all 4 values:



PYTHON CODE

```
import matplotlib.pyplot as plt
from matplotlib.lines import Line2D
import numpy as np
import math
import os
from pylab import *
from matplotlib import ticker
from matplotlib.ticker import ScalarFormatter
sformatter=ScalarFormatter(useOffset=True,useMathText=True)
sformatter.set\_scientific(True)
sformatter.set\_powerlimits((-2,3))

def phaseshiftgetter(betaa,kaval):
  deltak=-kaval+np.arctan(sin(kaval)/(cos(kaval)-(betaa/kaval)*sin(kaval)))
  return deltak

font = {'family' : 'serif',
        'weight' : 'normal',
        'size'   : 14}
plt.rc('font', **font)
plt.rc('text', usetex=False)
plt.figure(figsize=(6,5))
fig = plt.figure(1)
ax = fig.add\_axes([0.15,0.12,0.8,0.8])

Np=50000
delk=0.0001
```

```
delta=zeros(Np,dtype='float')
ka=zeros(Np,dtype='float')

betaa=0.5
for i in range(0,Np):
  ka[i]=delk*(i+0.5)
  kka=ka[i]
  delta[i]=phaseshiftgetter(betaa,kka)
  while delta[i] < 0.0:
    delta[i]=delta[i]+pi
plt.plot(ka,delta*180/pi,linewidth=3,color='k',label='\$\\beta a\$=0.5')

betaa=0.99
for i in range(0,Np):
  ka[i]=delk*(i+0.5)
  kka=ka[i]
  delta[i]=phaseshiftgetter(betaa,kka)
  while delta[i] < 0.0:
    delta[i]=delta[i]+pi
plt.plot(ka,delta*180/pi,linewidth=3,color='r',label='\$\\beta a\$=0.99')

betaa=1.01
for i in range(0,Np):
  ka[i]=delk*(i+0.5)
  kka=ka[i]
  delta[i]=phaseshiftgetter(betaa,kka)
  while delta[i] < 0.0:
    delta[i]=delta[i]+pi
plt.plot(ka,delta*180/pi,linewidth=3,color='g',label='\$\\beta a\$=1.01')

betaa=1.5
for i in range(0,Np):
  ka[i]=delk*(i+0.5)
  kka=ka[i]
  delta[i]=phaseshiftgetter(betaa,kka)
  while delta[i] < 0.0:
    delta[i]=delta[i]+pi
plt.plot(ka,delta*180/pi,linewidth=3,color='b',label='\$\\beta a\$=1.5')

ax.legend()

ax.tick\_params(axis='both', which='major', labelsize=14)
ax.set\_xticks(np.arange(0,6,1), minor=False)
ax.set\_xticklabels(np.arange(0,6,1), minor=False, family='serif')
ax.set\_xticks(np.arange(0,6,0.5), minor=True)
ax.xaxis.set\_major\_formatter(ticker.FormatStrFormatter('%0.1f'))
plt.xlim(0.0,5)

ax.set\_yticks(np.arange(-30,300,30), minor=False)
```

```
ax.set\_yticklabels(np.arange(-30,300,30), minor=False, family='serif')
ax.set\_yticks(np.arange(-30,300,10), minor=True)
plt.ylim(-5,185.0)

plt.xlabel('\$ka\$', fontsize=18, weight='normal')
plt.ylabel('\$\\delta\$ (degrees)',fontsize=18)
plt.savefig('phaseshift.pdf',format='pdf')
%#os.system('open -a Preview phaseshift.pdf')
plt.show()
quit()
```