your name(s) $\qquad$

Physics 851 Exercise \#4-Monday, Oct 4th, 2021
Consider a particle of mass $\boldsymbol{m}$ under the influence of the potential,

$$
V(x)=V_{0} \theta(-x)-\frac{\hbar^{2}}{2 m} \beta \delta(x-a), \quad V_{0} \rightarrow \infty, \beta>0 .
$$

A plane wave moving in the $-\hat{x}$ direction is reflected off the potential. $\operatorname{For}(x>a)$ the plane wave will have the form

$$
e^{-i k x}-e^{2 i \delta} e^{i k x}
$$

Here, $\boldsymbol{\delta}$ is referred to as the phase shift.

1. Find $\boldsymbol{\delta}$ as a function of $\boldsymbol{k a}$, and plot for $\boldsymbol{\beta a}=\mathbf{0 . 5}$ and for $\mathbf{0}<\boldsymbol{k a}<\mathbf{1 0}$. Because addition of $\boldsymbol{n} \boldsymbol{\pi}$ to the phase shift is arbitrary, translate all phase shifts to angles between zero and $\pi$.
2. Repeat for $\beta a=0.99,1.01,1.5$.
