your name(s)_____

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Physics 851 Exercise #3 - Monday, Sep. 27 2021

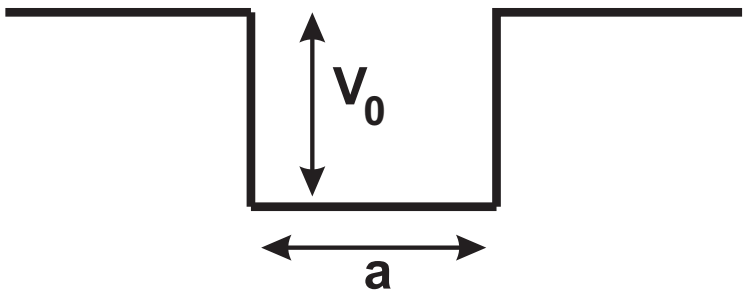
A beam of particles of N particles of mass m and momentum p has a wave function spread over a large length L,

$$\psi_p(x) = \frac{e^{ipx/\hbar}}{\sqrt{L}}.$$

While the beam is passing by the origin, a potential suddenly appears,

$$V(x) = \begin{cases} 0, & t < 0 \\ -V_0\Theta(x + a/2)\Theta(a/2 - x), & t > 0 \end{cases},$$

where Θ is a step function.



The depth of the potential is adjusted so that the ground state energy is $-V_0/2$.

Assume the normalized wave function of the ground state has the form,

$$\phi(x) = Z^{-1/2} \begin{cases} \cos(kx), & |x| < a/2, \\ Ae^{-q(|x|-a/2)}, & |x| > a/2 \end{cases}.$$

1. What are q and k in terms of V_0 and m?
2. What is A in terms of q?
3. What is Z in terms of q and a?
4. If there is a single particle, what is the probability it will fall into the ground state? Express your answer in terms of q, a, A and Z.
5. In terms of q, a, A, Z and the density (number per unit length), $\rho = N/L$, what is the average number of particles that will be in the ground state at large times?
6. For $t < 0$ how many states of momentum p are there per differential momentum, i.e. what is dN_{states}/dp?
7. For $t < 0$ what is the average occupancy of a momentum state with momentum p if the momentum distribution is proportional to $e^{-E/T}$?
8. Assuming the thermal distribution above, write an integral to express out how many particles are in the ground state for $t > 0$. (Don't perform the integral)