your name(s)____

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Physics 851 Exercise #2 - Monday, Sept. 20th

Neutral Kaon Oscillations: There are two kinds of neutral kaons one can make using down and strange quarks,

$$|K^0\rangle = |d\bar{s}\rangle, \;\; |\bar{K}^0\rangle = |s\bar{d}\rangle.$$

If it weren't for the weak interaction, the two species would have equal masses, and the Hamiltonian (for a kaon with zero momentum) would be

$$H_0 = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}.$$

However, there is an additional term from the weak interaction that mixes the states,

$$H_m = \begin{pmatrix} 0 & \epsilon \\ \epsilon & 0 \end{pmatrix}.$$

The masses of a neutral kaon are 497.6 MeV/c^2, without mixing, but after adding the mixing term the masses differ by 3.56μeV/c^2. The two eigenstates are known as K_S (K-short) and K_L (K-long), because they decay with quite different lifetimes.

1. What is ϵ?
2. If one creates a kaon in the K_0 state at time $t = 0$, find the probability it would be measured as a $\bar{K}^0$ as a function of time.
3. A beam kaons is created in the K_0 channel and has a kinetic energy of 600 MeV per kaon. Plot the probability that the kaon is in the K_0 state as a function of the distance traveled, x. Ignore the fact that the kaons decay.
4. Repeat (c), but take into account the decays. The states

$$|K_S\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle),$$
$$|K_L\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle),$$

known at K-short and K-long, represent the eigenstates of the Hamiltonian. The lifetime of a K_L is 51.2 ns, and the lifetime of the K_S is 0.0896 ns. Note that the wave function should be modified by the factor $e^{-t/(2\tau)}$ to take decays into account decays of lifetime τ.

FYI: If the above were exactly true, the K_S state would be even under CP while the K_L would be odd under CP. Here, CP is an operator that changes particles to anti-particles. If the particle-antiparticle symmetry were exact, the CP operator would commute with the Hamiltonian and the eigenstates of the Hamiltonian, K_S and K_L, would have to be eigenstates of CP. The K_S would then decay to two pions and the K_L could decay to three pions. However, there is an additional small CP violating term in the Hamiltonian (even smaller than the mixing term) which allows K_L to have a small probability of decaying to two pions. This was the first experimental laboratory observation of CP violation. CP violation is required to explain the preponderance of matter vs. anti-matter in the universe.