your name(s)_____

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Physics 851 Exercise #14 - Monday, Nov. 22

The Rutherford cross section for a charge e of wave number k scattering off a target charge Ze is

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} = \frac{Z^2e^4m^2}{(\hbar k)^4(1-\cos\theta)^2}.$$

Now, consider two charges, a positive charge Ze at the origin and a negative charge $-Ze$ at $a\hat{z}$.

1. What is the differential cross section?
2. What are the angles at which the cross section vanishes?
3. On a logarithmic plot, graph the differential cross section vs. θ for $ka = 1, 4, 10$. Scale the cross section by the factor $Z^2e^4m^2/(\hbar k)^4$.

Solution:
a) Multiply the cross section by f^2, with

$$\begin{aligned} f &= 1 - e^{iq_z a}, \quad \textbf{Note sign!} \\ |f|^2 &= 4\sin^2(q_z a/2), \\ \frac{d\sigma}{d\Omega} &= 4\left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} \sin^2(q_z a/2) \\ &= 4\frac{Z^2e^4m^2}{(\hbar k)^4(1-\cos\theta)^2}\sin^2[ka(1-\cos\theta)/2]. \end{aligned}$$

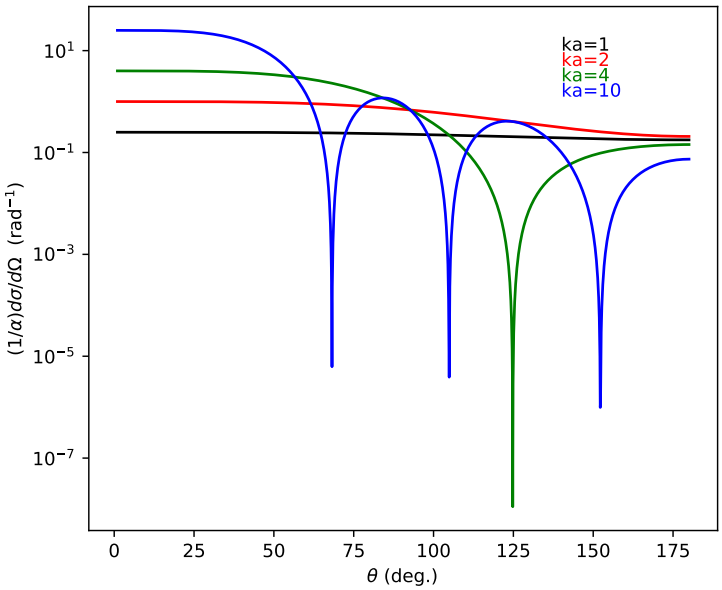
b)

$$ka(1-\cos\theta)/2 = n\pi, \;\; n = 1, 2, 3\cdots, \tag{0.1}$$

$$\theta = \cos^{-1}\left\{1 - \frac{2n\pi}{ka}\right\}$$

Note that $n = 0$ does not work because the Rutherford cross section goes to infinity in such a way that it cancels the zero.
c) Scaling the cross section by $\alpha = Z^2e^4m^2/(\hbar k)^4$,

$$\frac{1}{\alpha}\frac{d\sigma}{d\Omega} = \frac{4}{(1-\cos\theta)^2}\sin^2[ka(1-\cos\theta)/2]. \tag{0.2}$$


ka=1
ka=2
ka=4
ka=10
10^{1}
10^{-1}
10^{-3}
10^{-5}
10^{-7}
$(1/\alpha)d\sigma/d\Omega$ (rad^{-1})
0
25
50
75
100
125
150
175
θ (deg.)