your name(s) $\qquad$

Physics 851 Exercise \#11-Monday, Nov. 22nd
Consider a one-dimensional world where a particle of mass $m$ experiences the attractive potential,

$$
V_{0}(x)=-\frac{\hbar^{2}}{m b} \delta(x)
$$

A particle in the bound state of the well then experiences a small external potential,

$$
\begin{array}{r}
V_{p}(t)=v_{0} \cos \omega t, \\
\hbar \omega>\frac{\hbar^{2}}{2 m b^{2}} .
\end{array}
$$

1. What is the bound-state energy $\boldsymbol{B}$ of the original well (ignore the external potential)? If you know, or can look up the answer, just write it down.
2. What is the energy, $\boldsymbol{E}$, and wavenumber $\boldsymbol{k}$ of the liberated particles?
3. Again ignoring the small external potential, find the wave function where at large times (long after $\boldsymbol{V}_{\boldsymbol{p}}$ is turned off) there is an outgoing plane wave $e^{i k x} / \sqrt{L}$ with $k>0$, i.e. it moves in the positive $\boldsymbol{x}$ direction. For this boundary condition have an outgoing wave for $\boldsymbol{x}>\boldsymbol{0}$ and incoming waves for both $\boldsymbol{x}<\mathbf{0}$ and for $\boldsymbol{x}>\mathbf{0}$. This wave function describes that of a created particle with asymptotic momentum $\boldsymbol{k}$. At some large time ( $\boldsymbol{v t} \gg \boldsymbol{L}$ ), the incoming waves disappear and there is only an outgoing wave.
4. Calculate the overlap of the outgoing wave function

$$
\alpha(k) \equiv\left\langle k \mid \psi_{0}\right\rangle,
$$

where $|\boldsymbol{k}\rangle$ is the state described above and $\left|\boldsymbol{\psi}_{\mathbf{0}}\right\rangle$ is the bound state. Give your answer in terms of $\boldsymbol{k}$ and $b$.
5. What is the rate at which one liberates the particle?

