SUBJECT EXAM

PHYSICS 851/852, SPRING 1999

1. Consider a particle of mass m that feels an attractive one-dimensional delta function potential,

$$V(x) = -\beta\delta(x)$$

   (a) (5 pt.s) Derive the ground state energy.

   (b) (10 pt.s) Consider a particle in the ground state of the well. If the well suddenly dissolves, find the differential probability of observing an asymptotic momentum state p.

2. (10 pt.s) Express the state $|s = 1/2, \ell = 1, m_s = 1/2, m_\ell = 0\rangle$ as a linear combination of eigenstates of total angular momentum J and projection M.

3. Two types of spin-1/2 fermions, referred to as "*bob*"s and "*carol*"s, exist in a **TWO-DIMENSIONAL WORLD**. They may undergo reactions, *bob* $\leftrightarrow$ *carol* $+ \gamma$, where γ refers to a photon. The masses, m, of *bob*s and *carol*s are identical and the net density, $n = n_b + n_c$, is fixed. The *carol*s feel an additional attractive energy U, which lowers their energy relative to the *bob*s.

   (a) (10 pt.s) For an equilibrated system at zero temperature, what fraction of the particles are *bob*s? Give your answer in terms of n, m, U and $\hbar$.

   (b) (5 pt.s) Demonstrate that the fraction you gave as the answer above is dimensionless.

4. (a) (5 pt.s) Consider an operator A in the Schrödinger representation. Given the Hamiltonian, $H = H_0 + V$, write expressions for $A_H(t)$ and $A_{\rm int}(t)$ which are the Heisenberg and interaction representations of A.

   (b) (5 pt.s) Show that in the interaction representation, the evolution operator,

$$U(t) \equiv e^{iH_0t}e^{-iHt} \ ,$$

   satisfies the equality,

$$\langle\psi|e^{iHt}Ae^{-iHt}|\phi\rangle = \langle\psi|U^\dagger(t)A_{\rm int}(t)U(t)|\phi\rangle$$

   (c) (5 pt.s) Show that U satisfies the equation,

$$U(t_f - t_0) = 1 - \frac{i}{\hbar}\int_{t_0}^{t_f} dt' V(t')U(t' - t_0)$$

5. (15 pt.s) If an interaction has an explicit time dependence, $V_t = V\cos\omega t$, Fermi's golden rule becomes:

$$\Gamma_{f\to i} = \frac{2\pi}{4\hbar}|\langle f|V|i\rangle|^2 \left\{\delta(\epsilon_f - \epsilon_i - \hbar\omega) + \delta(\epsilon_f - \epsilon_i + \hbar\omega)\right\}$$

Consider a particle of mass m in the ground state of a delta function potential, with the wave function:

$$\psi_0(x) = \sqrt{k}e^{-k|x|}.$$

An oscillating electric field is added that contributes a term,

$$V_t = Fx\cos(\omega t),$$

to the Hamiltonian. The frequency, ω, corresponds to an energy greater than the binding energy of the well, $\hbar\omega > \hbar^2k^2/(2m)$.

Estimate the rate at which the particle is ionized using Fermi's golden rule.

6. (15 pt.s) Consider eigenstates of the hydrogen atom whose angular wave functions are described by ℓ and m_ℓ. Which of the following matrix elements equal zero? All other information about the eigenstate (e.g. spin and radial wave functions) are referred to by α and β

   (a) $\langle \alpha, \ell = 2, m_\ell = 0|\ r^2\ |\beta, \ell = 0, m_\ell = 0\rangle$

   (b) $\langle \alpha, \ell = 2, m_\ell = 0|\ x^2 + y^2\ |\beta, \ell = 0, m_\ell = 0\rangle$

   (c) $\langle \alpha, \ell = 3, m_\ell = 0|\ z\ |\beta, \ell = 0, m_\ell = 0\rangle$

   (d) $\langle \alpha, \ell = 3, m_\ell = 3|\ z^2\ |\beta, \ell = 3, m_\ell = 3\rangle$

   (e) $\langle \alpha, \ell = 3, m_\ell = 3|\ z^2\ |\beta, \ell = 3, m_\ell = 1\rangle$

7. An electron is placed in a constant magnetic field of strength B which lies along the z axis. Neglect the coupling of the spin to $\vec{B}$, and assume the electron is confined two-dimensionally to the $z = 0$ plane.

   (a) (5 pt.s) Show that when using a gauge such that $\vec{A}$ lies purely along the y axis, that the operator $P_y \equiv -i\hbar\partial/\partial y$ commutes with the Hamiltonian.

   (b) (5 pt.s) Given that a wave function $\phi_{p_y}(x, y)$ is an eigenstate of P_y with eigenvalue p_y, and is also an eigenstate of the Hamiltonian, write an expression for the ground state wave function $\phi_{0,p_y}(x, y)$. (Do not concern yourself with the normalization.) What is the energy of the ground state?

   (c) (5 pt.s) Find the degeneracy of the ground state if the dimensions of the surface are L_x and L_y. Express your answer in term of e, c, B, m, L_x and L_y.