SUBJECT EXAM

PHYSICS 851/852, MAY 3, 2000

1. (10 pt.s) A magnetic field at 60° to the z axis is applied with the time dependence,

$$B(t) = \begin{cases} 0, & t < 0 \\ B_0, & t > 0 \end{cases}$$

If an at-rest electron is initially in a spin-up state (up being defined relative to the z axis), find the probability of the electron being in the spin-down state as a function of time t.

2. Consider the $\ell = 1$ basis where $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ represent the $m_\ell = 1, 0$ and -1 eigenstates, respectively, of L_z.

   (a) (10 pt.s) Write down matrices to represent the operators L_z, L_x and L_y in this basis.

   (b) (5 pt.s) Write down the matrix components of the operator that represents a rotation by an angle ϕ about the z axis.

3. (15 pt.s) Four types of spin-1/2 fermions, referred to as *bob*s and *carol*s, *ted*s and *alice*s, exist in a **TWO-DIMENSIONAL WORLD**. The net charge is conserved.

$$Q = eQ_{bob} - eQ_{carol} + 2eQ_{ted} - 2eQ_{alice}.$$

The overall baryon number is also conserved.

$$N_b = N_{bob} + N_{carol} + 2N_{ted} + 2N_{alice}.$$

They may undergo any reactions, e.g. $2 \cdot bob + 2 \cdot carol \leftrightarrow ted + alice$, that conserve charge and baryon number. The masses of the various species are:

$$m_{bob} = m_{carol} = m, \quad m_{ted} = m_{alice} = 2m.$$

Consider an isolated system of volume V with a net baryon number $N_b \neq 0$ and zero net charge $Q = 0$ which is in the lowest energy state.

Find the numbers of each species, N_{bob}, N_{carol}, N_{ted} and N_{alice} as a function of m, N_b and V.

4. An electron is placed in a constant magnetic field of strength B which lies along the z axis. The electron also experiences an electric field E which lies along the $\cancel{y}$ x axis. Neglect the coupling of the spin to the magnetic field.

   (a) (5 pt.s) Write down a vector potential $\mathbf{A}(\mathbf{r}, t)$ with $\mathbf{A}$ being solely along the y axis that results in the electromagnetic field described above.

   (b) (5 pt.s) Write the Hamiltonian for an electron in the field described above.

   (c) (5 pt.s) Assuming the wave function is of the form $\psi(\mathbf{r}, t) = e^{ik_y y + ik_z z}\phi_{k_y,k_z}(x, t)$, write the wave equation for $\phi_{k_y,k_z}(x, t)$ where $\hbar k_y$ and $\hbar k_z$ are the eigenvalues of P_y and P_z.

5. A particle of mass m, moving in a **ONE-DIMENSIONAL** world, is confined to the ground state of an infinite square well,

$$0 < x < L.$$

At a time $t = 0$, the well is suddenly expanded to twice the original size.

$$0 < x < 2L.$$

(a) (5 pt.s) What is the probability that the particle will be found in the ground state of the new well?

(b) (5 pt.s) What is the expectation of the energy, $\langle\psi(t)|H|\psi(t)\rangle$, for times after the expansion of the square well.

FYI:

$$\begin{aligned}
\int_0^{\pi/2} d\theta \sin m\theta \sin n\theta &= \frac{-1}{2(m+n)}\sin(m+n)\frac{\pi}{2} + \frac{1}{2(m-n)}\sin(m-n)\frac{\pi}{2} \\
\int_0^{\frac{\pi}{2}} d\theta \cos m\theta \cos n\theta &= \frac{1}{2(m+n)}\sin(m+n)\frac{\pi}{2} + \frac{1}{2(m-n)}\sin(m-n)\frac{\pi}{2} \\
\int_0^{\frac{\pi}{2}} d\theta \sin m\theta \cos n\theta &= \frac{-1}{2(m+n)}\cos(m+n)\frac{\pi}{2} + \frac{-1}{2(m-n)}\cos(m-n)\frac{\pi}{2}
\end{aligned}$$

6. (15 pt.s) Consider eigenstates of the hydrogen atom whose angular wave functions are described by ℓ and m_ℓ. Using the Wigner-Eckart theorem and conservation of parity, determine which of the following matrix elements must equal zero? All other information about the eigenstate (e.g. spin and radial wave functions) are referred to by α and β

(a) $\langle\alpha, \ell = 2, m_\ell = 0|\ r^2\ |\beta, \ell = 2, m_\ell = 1\rangle$

(b) $\langle\alpha, \ell = 2, m_\ell = 0|\ x^2 + y^2\ |\beta, \ell = 1, m_\ell = 0\rangle$

(c) $\langle\alpha, \ell = 3, m_\ell = 0|\ z\ |\beta, \ell = 0, m_\ell = 0\rangle$

(d) $\langle\alpha, \ell = 3, m_\ell = 3|\ (x+iy)^2 + (x-iy)^2\ |\beta, \ell = 3, m_\ell = 3\rangle$

(e) $\langle\alpha, \ell = 3, m_\ell = 3|\ (x+iy)^2 + (x-iy)^2\ |\beta, \ell = 3, m_\ell = 1\rangle$

7. A particle of mass m moving through normal **THREE-DIMENSIONAL** space feels a spherically symmetric attractive potential,

$$V(r) = -\beta\delta(r - R),$$

where r is the distance from the origin.

(a) (10 pt.s) For fixed R and m, find the minimum strength of the potential β that results in the existence of a bound state. Express β_{min} as a function of $\hbar$, R and m.

(b) (5 pt.s) A spherical s wave scatters off the potential, with asymptotic form

$$\psi(r) \sim \frac{1}{r}\left(e^{-ikr} - e^{ikr+2i\delta}\right).$$

For the potential above (with arbitrary β), find the phase shift δ as a function of k.