FINAL EXAM, PHYSICS 851, FALL 2000

Thursday, December 14, 10:00 AM

This exam is worth 100 points

1. (10 pts) Consider two orthonormal states $|\uparrow\rangle$ and $|\downarrow\rangle$. If a particle is in the state $|\uparrow\rangle$ at time $t = 0$, and feels the interaction,

$$H = E\Theta(t)\left(|\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|\right),$$

find the probability for being in the state $|\downarrow\rangle$ as a function of time.

2. A particle of mass m is in the ground state of a one-dimensional harmonic oscillator characterized by frequency ω. At a time $t = 0$, the potential suddenly becomes zero and the particle becomes free.

   (a) (10 pts) What is the differential probability, dN/dp, of observing the particle with momentum p? Check that the normalization is correct,

$$i.e.\quad \int_{-\infty}^{\infty} dp\, \frac{dN}{dp} = 1.$$

   (b) (5 pts) What is the average energy of the emitted particle?

$$\bar{E} \equiv \int dp\, E_p \frac{dN}{dp}$$

3. A particle of mass m lies in the ground state of one-dimensional harmonic oscillator characterized by frequency ω. The particle also experiences a perturbative interaction V.

$$\begin{aligned} H &= H_0 + V \\ H_0 &= \frac{P^2}{2m} + \frac{1}{2}m\omega^2 X^2 \\ V &= \frac{m}{2}\beta X^2. \end{aligned}$$

   (a) (10pts) Find the correction to the ground state energy to second order in perturbation theory.

   (b) (5 pts) Write the exact correction due to the interaction.

4. (15 pts) Express the state $|\ell = 2, s = 1/2, J = 3/2, M = 3/2\rangle$ as a linear combination of eigenstates of L_z and S_z.

5. An electron is in an $\ell = 1$ state of a hydrogen atom. It experience a spin orbit interaction,

$$V_{\text{s.o.}} = \alpha \mathbf{L} \cdot \mathbf{S}$$

and also experiences an external magnetic field

$$V_b = -\mu \mathbf{B} \cdot (\mathbf{L} + 2\mathbf{S})\,.$$

(a) (5 pts) If the magnetic field is zero, what are the energy levels? Note the degeneracy of each level.

(b) (5 pts) If the field is non-zero but the spin-orbit coupling is neglected ($\alpha = 0$), what are the energy eigenvalues? Again, note the degeneracy of each level.

(c) (10 pts) When $\alpha \neq 0$, $B \neq 0$, and $\vec{B}$ points along the z axis, which of the following operators commute with the Hamiltonian. Yes-or-no answers are fine and no credit is given for wrong answers with good reasoning. (Note: $\vec{J} \equiv \vec{L} + \vec{S}$)

i. $|\vec{J}|^2 = J_x^2 + J_y^2 + J_z^2$.

ii. J_z

iii. L_z

iv. $|\vec{S}|^2$

v. $\vec{L} \cdot \vec{S}$

6. (25 pts) A particle of mass M is in the first excited state of a three-dimensional harmonic oscillator characterized by frequency ω. It can decay to the ground state via the emission of a *sally* particle which is massless and spinless. The matrix element responsible for the decay is

$$\langle k, 0|H_{\text{int}}|m\rangle = \beta \vec{k} \cdot \int d^3r\, \frac{e^{i\vec{k}\cdot\vec{r}}}{\sqrt{V}} \left[\psi_m^*(\vec{r})\vec{\partial}\psi_0(\vec{r}) - \psi_0^*(\vec{r})\vec{\partial}\psi_m(\vec{r})\right],$$

where $\hbar\vec{k}$ is the momentum of the outgoing *sally* particle. Find the differential decay rate, $d\Gamma_m/d\Omega$, where Ω refers to the direction of the outgoing *sally*, and m refers to the angular momentum projection of the initial state. For each value of m describe the angular distribution in terms of the polar angle θ and the azimuthal angle ϕ.

Potentially useful information:

$$\psi_0(\vec{r}) = \frac{1}{\pi^{3/4}a^{3/2}} e^{-r^2/(2a^2)}, \quad a^2 = \frac{\hbar}{M\omega} \tag{1}$$

$$\psi_m(\vec{r}) = \hat{m} \cdot \vec{r} \frac{\sqrt{2}}{a} \psi_0(\vec{r}) \tag{2}$$

$$\hat{m} = \frac{\hat{x} \pm i\hat{y}}{\sqrt{2}}, \ \hat{z} \ \text{ for } m = \pm 1, 0 \tag{3}$$

$$E = \hbar k c, \text{ for a massless particle.} \tag{4}$$