your name(s)____

Physics 841 Quiz #7 - Friday, Mar. 24

You may work in groups of 3 (no more than one person from previous group)

Open book, open notes, open mouth

Consider a simple model of the universe where the expansion velocity for cosmological purposes is $\vec{v} = \vec{r}/t$. This corresponds to a "flat" universe with gravitational effects ignored. All matter starts at a point (the origin) and there is no acceleration for any fluid element. Observer A is moving with the source, and records light being emitted at a time $\tau_0 = 10^5$ years after the birth of the universe. A second observer, B, records the light moving past at a time $\tau = 1.4 \times 10^{14}$ years after the beginning of the universe. Both A and B are at rest relative to the neighboring expanding matter. If observer A records the frequency of the emitted light as being f_0, find the frequency f of the recorded light according to observer B.

Some Help: the time measured by the co-moving observer, τ, is related to the time measured by a different observer with velocity v by the relations:

$$\tau = \frac{t}{\gamma} = t\sqrt{1-v^2} = t\sqrt{1-r^2/t^2} = \sqrt{t^2-r^2}.$$

Solutions:

Let $t_0 = \tau_0$ and t be the times of emission and measurement according to an observer in the frame of the source, and r be the distance, according to this observer, at which the source is observed. Observer B has velocity v in this frame.

$$\begin{aligned} r &= vt = c(t-t_0), \\ t_0 &= t(1-v). \end{aligned}$$

(I cut out the cs) Divide this equation by

$$\tau = t\sqrt{(1-v^2)},$$

to obtain

$$\frac{\tau_0}{\tau} = \sqrt{1-v}1+v.$$

Thus the red shift is

$$\frac{\omega}{\omega_0} = \frac{\tau_0}{\tau} = 7.14 \times 10^{-6}.$$

Aside: For the sake of curiosity, one can solve for v/c.

$$\begin{aligned} \sqrt{\frac{1-v}{1+v}} &= \frac{\tau_0}{\tau}, \\ 1-v &= \frac{\tau_0^2}{\tau^2}(1+v), \\ v &= \frac{1-\tau_0^2/\tau^2}{1+\tau_0^2/\tau^2} \\ &= 1.0 - 1.0204 \times 10^{-10} = 0.99999999989868 \end{aligned}$$