Quiz 12
(Practice Exam Part I)
Solutions

1 a) NO, because $B^2 - E^2$ = Lorentz invariant

1b) yes

1c) Choose $A_0 = -E_x x$

$A_y = -B_z x$

Boost along y direction

$A_0' = \gamma(-E_x x + v B_z x)$

$A_y' = \gamma(-B_z x + v E_x x)$

v = velocity of observer

choose $v = E_x / B_z$ is small, so $\gamma \to 1$

$A_0' = 0, \quad A_y' = \gamma(-B_z x + v E_x x)$

$$B_z' = B_z\left(1 - \frac{E_x^2}{B_z^2}\right)\frac{1}{\sqrt{1 - E_x^2/B_z^2}}$$

$$= B_z\sqrt{1 - E_x^2/B_z^2}$$

In primed frame, (non-rel motion)

$v_{0y}' = -(E_x/B_z), \quad v_{0x}' = 0$

$e v' B' = m v'^2/R, \quad R = \frac{m v'}{e B'}$

$R = \frac{m(E_x/B_z)}{e B_z}$, $\quad v_y' = -\frac{E_x}{B_z}\cos\left(\frac{e B_z t}{m}\right)$

non-rel.

$\omega = v'/R = \frac{e B}{m}$ $\quad v_x' = \boxed{\frac{E_x}{B_z}\sin\left(\frac{e B_z t}{m}\right) = V_x}$

$$\boxed{V_y = \frac{E_x}{B_z}\left(1 - \cos\left(e B_z t/m\right)\right)}$$

2a) $\dfrac{(x_a - x_b) \cdot P_c}{M_c}$

2b) $\dfrac{(x_a - x_b) \cdot P_r}{M_r}$

3) a) Gauss's Law $\rightarrow$ $E_z = 4\pi\sigma$ = const.

b) $T^{\alpha \neq \beta} = 0$

$$T^{oo} = \frac{1}{8\pi} E_z^2 ,$$

$$T^{xx} = \frac{1}{8\pi} E_z^2 = T^{yy}$$

$$T^{zz} = -\frac{1}{8\pi} E_z^2$$

$$T^{\alpha}{}_{\alpha} = T^{oo} - T^{xx} - T^{yy} - T^{zz} = 0$$

c) $U = \frac{1}{8\pi} E_z^2 \cdot A \cdot v t$

$$\frac{dU}{dt} = \frac{1}{8\pi} A E_z^2 = \text{Power} \qquad = -T_{zz} A$$

4 a) False, not if $\vec{E}\cdot\vec{B} \neq 0$

b) False.

c) False, B = pseudo vector

d) True

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5) L_x -

only about center of motion

cylindrical coordinates about x axis.

You can rewrite vector potential as

$$\vec{A} = \frac{1}{2} B r \hat{\varphi}$$

Lagrangian will have no dependence on φ, but the conserved quantity from Noether's theorem is

$$L_x + \epsilon B r^2$$

and L_x is only constant if I.C. make circular motion about origin.