- Find the instantaneous power radiated in the dipole approximation as a function of the energy $E(t)$

Begin with the standard equation for power radiated by a point charge accelerating perpendicularly to it's velocity.

$$P = \frac{2}{3c} e^2 \dot{\beta}^2 \tag{1}$$

Remembering that $\beta = \frac{v}{c}$ if working in units where $v \neq c$ and applying the Lorentz force law $F = q\frac{\vec{v}}{c} \times \vec{B}$

$$\dot{\beta} = \frac{evB}{mc^2} \tag{2}$$

$$P = \frac{2e^4 v^2 B^2}{3m^2 c^5} \tag{3}$$

$$\tag{4}$$

This can be cleaned up a bit by recognizing the cyclotron frequency $\omega_c = \frac{eB}{mc}$ which is convenient because it can be calculated in any sane system of units to deal with the B field. Letting $E = \frac{1}{2}mv^2$ for a classical particle,

$$P = \frac{4e^2 {\omega_c}^2}{3(mc^2)c} E \tag{5}$$

- Find the instantaneous power radiated in the dipole approximation as a function of the energy $E(t)$

Since $P = \frac{dE}{dt}$ is the energy *lost* by the particle

$$-\frac{dE}{E} = \frac{4e^2 {\omega_c}^2}{3(mc^2)c} dt$$

$$E = E_0 e^{-\frac{4e^2 {\omega_c}^2}{3(mc^2)c} t}$$

- Find the amount of time necessary for the electron's energy to decrease by a factor of 2

Starting with the expression for energy and expressing the result in terms of $\hbar$ and the fine structure constant to simplify numerical calculations

$$E = E_0 e^{-\frac{4e^2 {\omega_c}^2}{3(mc^2)c} t} \tag{6}$$

$$\frac{1}{2} = e^{-\frac{4e^2 {\omega_c}^2}{3(mc^2)c} t} \tag{7}$$

$$\ln 2 = \frac{4e^2 {\omega_c}^2}{3(mc^2)c} t \tag{8}$$

$$\frac{t}{\tau} = \frac{\omega_c t}{2\pi} = \frac{3 \ln 2 (mc^2) c}{8\pi e^2 \omega_c} \tag{9}$$

$$\frac{t}{\tau} = \frac{3 \ln 2}{8\pi} \frac{mc^2}{\alpha \omega_c \hbar} \tag{10}$$

This dimensionless result gives the time for the power to decrease by a factor of two relative to the period of its cyclotron motion, and for $B = 1T$ works out to $\frac{t}{\tau} \approx 4 \times 10^{11}$ so the electron radiates power very slowly compared to its motion.