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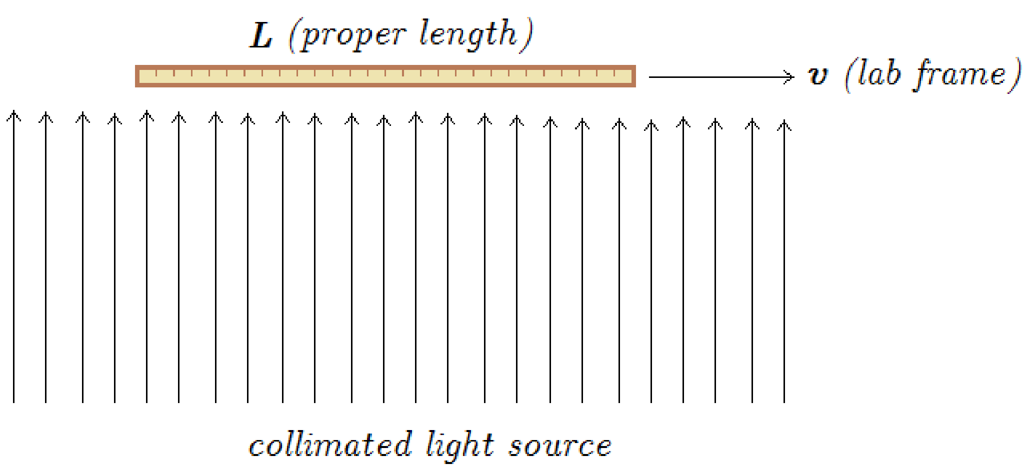
Imagine a ruler of length L (when at rest) moving at speed v with respect to our laboratory frame, where the direction of v is along the length of the ruler. There is also a light source set up such that it emits a flash of light (collimated perpendicular to the ruler) that reaches both ends of the ruler simultaneously in the laboratory frame; as a result of this light flash, there will be a shadow of the ruler on the wall. Find what the length of the shadow will be in both the laboratory's and the ruler's reference frame. Neglect the effects of diffraction around the edges of the ruler.



SOLUTION:

In the laboratory frame, the collimated light hits the ruler, which is length contracted by the factor $\gamma = (1 - v^2)^{-1/2}$. Since it's collimated, the shadow behind it will be the same length, meaning <u>it's $\boldsymbol{L/\gamma}$.</u>

In the ruler's frame, the light will hit both ends at different times, but the shadow will still end up being the same length as the ruler, since light that is collimated in one frame will be collimated in another. This can be shown by Lorentz transform—if $x = 0$ at the left side of the ruler and $x = L/\gamma$ is the right side, and both occur at t=0 (all in the lab frame), then:

$$\begin{pmatrix} t_L' \\ x_L' \end{pmatrix} \begin{pmatrix} \gamma & -\gamma v \\ -\gamma v & \gamma \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

and

$$\begin{pmatrix} t_R' \\ x_R' \end{pmatrix} \begin{pmatrix} \gamma & -\gamma v \\ -\gamma v & \gamma \end{pmatrix} \begin{pmatrix} 0 \\ L/\gamma \end{pmatrix} = \begin{pmatrix} -\gamma v L \\ \gamma L/\gamma \end{pmatrix} = \begin{pmatrix} -\gamma v L \\ L \end{pmatrix}$$

Then the two are still separated by $\Delta x' = x_R' - x_L' = L$, and the collimated light projects that distance onto the shadow: <u>it's $\boldsymbol{L}$.</u>