

Solution:

In Earth frame:

Event A, sending message A(t,0); units of day, c=1;

In Astronaut frame:

Event A, sending message A'(1.25t,-0.75t); units of day, c=1;

$$t' = \gamma * t - \gamma \beta * x = 1.25 * t$$

$$x' = \gamma * x - \gamma \beta * t = -0.75 * t$$

Event B, happy birthday B'(10,0);

Since the signal travels at the speed of light, Event B should be on the light cone spanned by event A,

$$\frac{1.25t - 10}{-0.75t} = 1, c = 1;$$

t=5 days.