Statistical Mechanics, Subject Examination
September 6, 2006
Time: 3 hours (6:00-9:00 pm)

Student Number:

There are five problems. Please use different sheets for each problem. Do not write your name. Write your student number on every page. Please show your work neatly so that partial credits can be given.

$$I_n = \int_0^\infty \frac{x^{n-1}}{e^x - 1} dx = \Gamma(n)\varsigma(n),$$

where $\Gamma(n)$ is the Gamma function and $\varsigma(n)$ is the Riemann zeta function, some of whose values are given below.

$$\varsigma(2) \approx 1.645, \varsigma(3) \approx 1.202, \varsigma(4) \approx 1.082$$
$$\varsigma(5) \approx 1.037, \varsigma(6) \approx 1.017$$

$$\Gamma(n) = (\text{n-1})!$$

$$\int_0^\infty e^{-bx^2} dx = \sqrt{\pi}/2b \quad \int_0^\infty x^n \, e^{-\mu x} dx = n!/\mu^{n+1}$$

Volume of a n-dimensional sphere of radius R: $V_n(R) = \dfrac{\pi^{n/2}}{(n/2)!} R^n$

For values of the fundamental constants please use the attached sheet.

Problem # 1 (20) points

A graphite surface is exposed to an atmosphere of argon gas (ideal) at temperature T and pressure P. Some of the argon atoms get adsorbed on the graphite surface. Once adsorbed (binding energy of each atom is E_B) they behave like a 2-dimensional classical ideal gas. Derive an expression for the density of adsorbed atoms as a function of temperature T and P. Discuss the pressure dependence for $k_B T << E_B$

Problem # 2 (20) points

The van der Waals gas is described by the formula

$$P = nRT/(V\text{-}b) - a/V^2$$

(a) Derive a general formula for the Joule-Thomson coefficient $(\partial U/\partial V)_T$ in terms of T, P and $(\partial P/\partial T)_V$. Calculate the Joule-Thomson coefficient for the van der Waals gas.

(b) A van der Waals gas expands from volume V_1 to V_2, but no work is done and no heat is added or subtracted. Using your result in part (a), derive a formula for the change in temperature in terms of V_1, V_2 and C_V, the specific heat at constant volume. Similarly, derive a formula for the change in entropy in terms of T_1, T_2, V_1, V_2 and C_V.

Problem # 3 (20) points

Consider a quantum system consisting of two non-interacting particles. Each particle can be in one of two <u>orbital</u> states for which the energies are $+\varepsilon_0$ and $-\varepsilon_0$. The system is at temperature T.

(a) If the particles are <u>distinguishable</u> and each with a spin of S=½, what is the total number of possible states for the system?

(b) If the particles are <u>indistinguishable fermions</u> with a spin of S=½, what is the total number of possible states for the system?

(c) If the particles are <u>indistinguishable bosons</u> with a spin S=2, what is the total number of possible states for the system?

(d) Find the partition function in terms of ε_0 and temperature T for the system describe in part (c).

(e) Find the average energy per particle for the system described in part (c).

Problem #4 (20 points)

Consider a white dwarf star of density 10^9 g/cc and temperature 10^7 K composed only of helium (^3He); for comparison, the density of water is approximately 1 g/cc at room temperature. The virial theorem for a gravitational potential is $\langle T \rangle = -0.5 \langle U \rangle$, where T and U are the kinetic energy and the gravitational potential energy, respectively. The mass of the proton/neutron and electron are 1.67×10^{-27} kg and 9.11×10^{-31} kg, respectively.

(a) Estimate the mean distance between the helium nuclei.

(b) Derive a formula for the mean energy of the electrons, assuming the extreme relativistic limit. Why is this limit justified?

(c) Do likewise for the helium nuclei. Is the extreme relativistic limit justified? How does their contribution to energy compare to that of the electrons?

(d) Use the virial theorem to estimate the mass of the star. What happens if the stellar mass is less than or greater than this value? Why does the virial theorem break down?

Problem #5 (20 points)

Consider a system of extreme relativistic gas of non-interacting bosons (energy $\varepsilon(\vec{p}) = c\,p$ where $\vec{p}$ is the momentum of a particle) of density n. Derive an equation for the Bose Einstein condensation temperature T_c for this system and discuss how T_c depends on n.