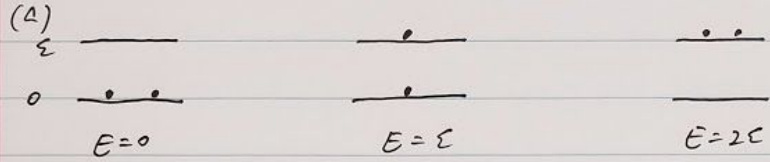
(a)



For each configuration, we have degeneracy $(m_1, m_2) = (1,1)$ $(0,0)$, $(-1,-1)$ $(1,0)$, $(0,1)$, $(-1,0)$ $(0,-1)$, $(-1,1)$ and $(1,-1)$

However, Since bosons are indisguishable, thus for $E=0$ and $E=2\varepsilon$

$(1,0)$ $(1,-1)$ $(-1,0)$ are the same
$(0,1)$ $(-1,1)$ $(0,-1)$

Therefore, degeneracy's for $E=0$ and $E=2\varepsilon$ are 6. and for $E=\varepsilon$ is 9.

$$Z_c = \sum g_i e^{-\beta\varepsilon_i} = 6 + 9e^{-\beta\varepsilon} + 6e^{-2\beta\varepsilon}$$

(b)

$$\langle E\rangle = -\frac{\partial}{\partial\beta}\ln Z_c = -\frac{-9\varepsilon e^{-\beta\varepsilon} - 12\varepsilon e^{-2\beta\varepsilon}}{6+9e^{-\beta\varepsilon}+6e^{-2\beta\varepsilon}} = \frac{9\varepsilon e^{-\beta\varepsilon} + 12\varepsilon e^{-2\beta\varepsilon}}{6+9e^{-\beta\varepsilon}+6e^{-2\beta\varepsilon}}$$

$\langle E\rangle_{T\to 0} = 0$ $\qquad$ $\langle E\rangle_{T\to\infty} = \varepsilon$

(c)

$$S = \ln Z + \beta\langle E\rangle = \ln(6+9e^{-\beta\varepsilon}+6e^{-2\beta\varepsilon}) + \frac{9\beta\varepsilon e^{-\beta\varepsilon} + 12\beta\varepsilon e^{-2\beta\varepsilon}}{6+9e^{-\beta\varepsilon}+6e^{-2\beta\varepsilon}}$$

$S_{T\to 0} = \ln(6)$

$S_{T\to\infty} = \ln(21)$