

Physics 831 Quiz #7 - Monday, Nov. 4, 2013

YOUR NAME: _____

Consider a two-dimensional array of N coupled two-dimensional harmonic oscillators, i.e., the oscillators only move in the $x - y$ plane.

1. At low temperature, the specific heat per oscillator can be expressed as:

$$C_V = \frac{dE}{dT} = \alpha T^n.$$

What is the power n ? _____.

2. What is the specific heat at high temperature? _____

3. If one doubles the speed of sound in (1) the parameter α will:

- (a) quadruple
- (b) double
- (c) increase by $\sqrt{2}$
- (d) stay the same
- (e) fall by $1/\sqrt{2}$
- (f) fall by $1/2$
- (g) fall by $1/4$.

4. (Extra Credit) Assume the oscillators each have mass m , are arranged in a square lattice, and are coupled by springs of spring constant k . Derive the speed of sound in terms of m , k , and the number of oscillators per area, ρ .

5. Show that in the mean field approximation of the Ising model the susceptibility,

$$\chi \equiv \frac{d\langle\sigma\rangle}{dB},$$

becomes

$$\chi = \frac{(1 - \langle\sigma\rangle^2)\mu}{T - T_c + \langle\sigma\rangle^2 T_c}.$$

Begin with the expressions: $\langle\sigma\rangle = \tanh(\beta q J \langle\sigma\rangle + \beta \mu B)$ and $T_c = qJ$.