1. (10 pts) Beginning with the fundamental thermodynamic relation,

$$
d S=\beta d E-\beta \mu d Q+\beta P d V
$$

derive the following Maxwell relation

$$
\left.\frac{\partial E}{\partial S}\right|_{\mu, V}=T-\left.\mu \frac{\partial T}{\partial \mu}\right|_{S, V} .
$$

2. ( 10 pts , extra credit) In Sec. 1.9 of the lecture notes, we showed the force acting on a particle,

$$
f=-\left.\frac{\partial E}{\partial x}\right|_{N, S, V}
$$

could equivalently be written as

$$
f=-\left.\frac{\partial F}{\partial x}\right|_{N, T, V}
$$

where $F=E-T S$ was the Helmholtz free energy. Show that one can extend this to

$$
-\left.\frac{\partial F}{\partial x}\right|_{N, T, V}=-\left.\frac{\partial(E-T S-\mu N)}{\partial x}\right|_{\mu, T, V}
$$

You may wish to remember that

$$
\mu=\frac{\partial F(N, T, V)}{\partial N}
$$

Just as an aside, $E-T S=-T \ln Z_{\text {can. }}$, and $E-T S-\mu N=-T \ln Z_{\text {G.C. }}$.

