Physics 831 Quiz #2 - Friday, October 2

1. Consider a **MASSLESS**($\epsilon_p = p$) three-dimensional gas of spinless bosons which is kept at temperature T. Solve for the density of Bose Condensation, $\rho_c(T)$. You can set $\hbar$ and $c = 1$ to save ink, and express sums in terms of Riemann-Zeta functions, $\zeta(n) \equiv \sum_k k^{-n}$.

2. Consider the equation of state

$$P = \rho T e^{\rho/\rho_0} - a\frac{\rho^2}{\rho_0}.$$

(a) Derive an expression for the energy per particle, E/N, as a function of the temperature T, the density ρ, and the parameters ρ_0 and a. Start your derivation with the Maxwell relation,

$$\left.\frac{\partial E}{\partial V}\right|_{\beta,N} = -\left.\frac{\partial(\beta P)}{\partial \beta}\right|_{V,N}$$

Continuing with the Eq. of state,

$$P = \rho T e^{\rho/\rho_0} - a\frac{\rho^2}{\rho_0}.$$

(b) Find the critical density, ρ_c, the critical temperature T_c and the critical pressure P_c. State your answers in terms of a and ρ_0.