Physics 831 Quiz #11 - Friday, Nov. 21

YOUR NAME:______________________________

1. Consider the following form for the free-energy density for a spin system where the σ can be between -1 and +1:

$$\begin{aligned} f(\sigma,T) &= \rho_0 \left\{ \mathcal{V}(\sigma,T) + \frac{\kappa}{2}(\nabla\sigma)^2 \right\}, \\ \mathcal{V}(\sigma,T) &= -\frac{1}{2}J\sigma^2 + T\frac{(1+\sigma)}{2}\ln\left(\frac{1+\sigma}{2}\right) + T\frac{(1-\sigma)}{2}\ln\left(\frac{1-\sigma}{2}\right). \end{aligned}$$

Fill in either *increase* or *decrease* for the following statements concerning the surface energy between a region where the spins have minimized f with $\sigma > 0$ and a separate region where $\sigma < 0$.

(a) The surface energy will ____________ if κ is increased.

(b) The surface energy will ____________ if J is increased.

(c) The surface energy will ____________ if T is increased (but kept less than T_c).

2. Two phase transitions have different critical exponents. You can safely conclude, (circle all correct answers)

- They come from different universality classes
- They have different dimensionalities
- They have different order parameters

3. In a two-dimensional ferromagnetic substance, spins become aligned when the temperature is lowered below the critical temperature, even though the material is in a field-free region. This is an example of: (circle all correct answers)

- spontaneous symmetry breaking
- explicit symmetry breaking
- breaking a continuous symmetry
- breaking a discrete symmetry

YOUR NAME:______________________________

4. Assume the free energy for a complex field in ONE dimensions is given by:

$$F = \int dx \frac{1}{2} \left(A\,|\phi|^2 + \kappa\,|\partial_x \phi|^2 \right).$$

Define the correlation Γ as

$$\Gamma(x) \equiv \langle \phi^*(0)\phi(x) \rangle.$$

Fourier transforms in one dimensions are defined by:

$$\tilde{\phi}_k \equiv \frac{1}{\sqrt{L}} \int dx\; e^{ikx}\phi(x), \qquad \phi(x) = \frac{1}{\sqrt{L}} \sum_k e^{-ikx}\phi_k.$$

(a) Calculation $\Gamma(x)$.

(b) What is the critical exponent ν?
The correlation length ξ behaves as $\xi \sim t^{-\nu}$ as $t = (T - T_c)/T_c \to 0$.

YOUR NAME:______________________________

5. The grand canonical partition function for a system of charged particles,

$$Z = \mathrm{Tr} e^{-\beta H + \beta\mu Q},$$

is given by:

$$\ln Z = A(T)\cosh(\mu/T).$$

In terms of μ, T, A, and derivatives of A,

(a) What is the average charge $\langle Q \rangle$?

(b) What is the fluctuation of the charge $\langle Q^2 - \langle Q \rangle^2 \rangle$?