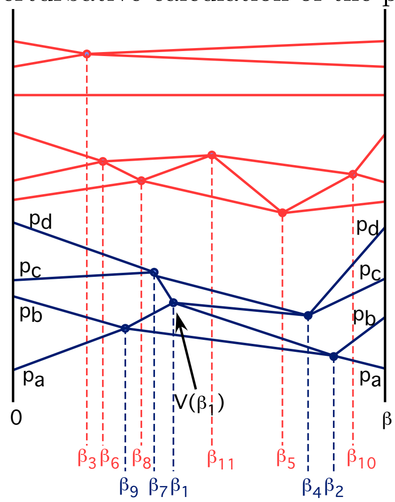
Physics 831 Quiz #10 - Friday, Nov. 14

YOUR NAME:________________________________

1. The diagram represents a perturbative calculation of the partition function.



   (a) Consider the connected diagram involving $p_a \rightarrow p_d$. When calculating the pressure, this diagram contributes to order $n =$________ in perturbation theory.

   (b) When performing a virial expansion (see expansion below), the lowest m for which this contributes to A_m is ________.

2. Consider a virial expansion for a non-relativistic ONE-dimensional gas of spin-zero bosons of mass m at temperature T,

$$\frac{P}{\rho T} = 1 + \sum_{m=2}^{\infty} A_m \left(\frac{\rho}{\rho_0}\right)^{m-1}, \quad \rho_0 \equiv \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} dp\; e^{-p^2/2mT}.$$

Ignoring interactions between the particles, calculate A_2. Here ρ is the number per unit length. Begin with the expression for the one-dimensional "pressure", and the density

$$\frac{PL}{T} = \ln Z = \frac{L}{2\pi\hbar}\int_{-\infty}^{\infty} dp\; \ln\left(\frac{1}{1 - e^{-\beta(p^2/2m-\mu)}}\right),$$

$$\rho = \frac{1}{L}\frac{\partial}{\partial \beta\mu}\ln Z.$$

3. Consider the state:

$$|\eta\rangle = e^{(\eta a^\dagger - \eta^* a)}|0\rangle.$$

Find the overlap $\langle 0|\eta\rangle$.
Hint: you may want to use the Campbell-Baker-Hausdorff lemma – If operators A and B commute to a number,

$$e^{A+B} = e^A e^B e^{-[A,B]/2}$$