Physics 831 Quiz #7 - Friday, Oct. 19

1. A box has a gas of oxygen molecules (mass-32) and hydrogen molecules (mass-2) in equal number densities. Particles are radiated through a small hole in the box for a short time. The ratio of emitted oxygen per emitted hydrogen molecules (N_O/N_H) is:

   a) 1/256
   b) 1/16
   c) 1/4
   d) 1 to 1
   e) 4
   f) 16
   g) 256

   Assume all phase space densities are sufficiently low to warrant Boltzmann statistics (not Bose or Fermi).

2. Consider a one dimensional box of length L, $0 < x < L$, filled with a liquid into which perfume molecules are dissolved, where the initial density, ρ_0, of perfume molecules is uniform throughout the box. At time $t = 0$, an electrified grid is turned on at the surface that captures all perfume molecules that happen to wander into the boundary ($x = 0$ or $x = L$)of the box. The molecules move according to the diffusion equation,

$$\frac{\partial \rho}{\partial t} = D\frac{\partial^2 \rho}{\partial x^2},$$

   where D is the diffusion constant describing the motion of the perfume molecules.

   (a) Find the Fourier coefficients, $A_m(t = 0)$, such that the initial density is expressed as a sum:

$$\rho(x, t) = \rho_0 \sum_{m=1,\infty}^{\infty} A_m(t) \sin(m\pi x/L).$$

   (b) Using the diffusion equation, solve for the time dependence of the coefficients $A_m(t)$.

Fun Facts to know and tell: For a function F defined between $x = 0$ and $x = L$,

$$\begin{aligned} F(x) &= \sum_{m=1}^{\infty} F_m \sin(m\pi x/L), \\ F_m &= \frac{2}{L}\int_0^L F(x) \sin(m\pi x/L). \end{aligned}$$
