Physics 831 Quiz #4 - Friday, Sep. 28

1. Beginning with:

$$dE = TdS - PdV + \mu dN,$$

derive the Maxwell relation:

$$\left.\frac{\partial S}{\partial V}\right|_{T,N} = \left.\frac{\partial P}{\partial T}\right|_{V,N}.$$

2. For a non-relativistic three-dimensional gas at low-density, show that the entropy per particle is:

$$\frac{S}{N} = \frac{5}{2} + \ln\left\{\frac{(mT)^{3/2}V}{(2\pi)^{3/2}\hbar^3 N}\right\}$$

Begin with:

$$\langle E/N\rangle = (3/2)T, \quad S = \ln Z_C + \beta\langle E\rangle, \quad Z_C(N,T) = \frac{z^N}{N!},$$

where z is the canonical partition function of a single particle of mass m in a volume V.

3. Using the information above, derive an expression for S/N in terms of V, N and T for the Van der Waals equation of state,

$$P = \frac{\rho T}{1 - \rho/\rho_s} - a\rho^2.$$

Write your result in a form that illustrates how S/N depends on the "excluded" volume.

4. Consider the isotherm (fixed temperature) on the $P - V$ diagram below. List all pairs of points that coexist at equilibrium.