Physics 831 Quiz #9 - Friday, Nov. 17

YOUR NAME:_____

1. Consider an independent spin whose values are either $\sigma_i = +1 \text{or} -1$. Beginning with the definition of entropy,

$$S = -\sum_\ell p_\ell \ln p_\ell,$$

where p_ℓ is the probability of a given level ℓ being occupied:

   (a) Find S in terms of $\langle\sigma\rangle$.

   (b) Assuming the spins have an interaction energy per spin,

$$E/N = -\frac{J}{2}\langle\sigma\rangle^2,$$

   Derive an expression for $\langle\sigma\rangle$ that minimizes the free energy, $F = E - TS$.

   (c) How does your expression compare to the one given in the notes, $\langle\sigma\rangle = \tanh(\beta J\langle\sigma\rangle)$. If different, explain what physical assumptions are driving the difference.

2. Suppose one has calculated a partition function,

$$Z = \text{Tr } e^{-\beta H}, \quad H = SCF(T) - \mu\vec{B}\cdot\vec{S},$$

where SCF is some complicated function of the temperature and $\vec{S}$ is the net spin of the system. Further assume that after performing all the fancy calculations that

$$\ln Z = N\left\{a_1 \ln[a_2 \cosh(a_3\beta\mu B)]\right\},$$

where a_1, a_2 and a_3 are functions of the temperature, and N is the number of sites.

   (a) Find the average spin per site as a function of β, μB, a_1, a_2, and a_3

   (b) In terms of a_1, a_2, a_3 and β, what is the average spin per site for

      i. $B = 0$

      ii. $B = \infty$