YOUR NAME:\_\_\_\_

- 1. Consider two identical alpha particles (which are bosons with spin 0). They occupy two single-particle levels with energies, 0 and  $\epsilon$ .
  - (a) What is the probability that the system is at the lowest possible total energy as a function of the temperature T?
  - (b) What is the average energy when T = 0?
  - (c) What is the average energy when  $T = \infty$ ?
  - (d) What is the entropy when T = 0?
  - (e) What is the entropy when  $T \to \infty$ ?
- 2. Beginning with the fundamental relation,

$$TdS = dE + PdV - \mu dN,$$

derive the relation,

$$\left. \frac{\partial E}{\partial N} \right|_{P,S} = \mu - P \left. \frac{\partial \mu}{\partial P} \right|_{N,S}$$

3. Consider a Hamiltonian for a particle moving in one dimension,

$$H = \sqrt{m^2 + p^2} - A \ln(x/x_0) + Bx, \quad A > 0, B > 0.$$

which confines the particle to  $0 < x < \infty$ . Using the generalized equipartition theorem, or the virial theorem, find the average value of  $\langle x \rangle$  as a function of T.

- 4. Circle the systems that allow Bose Condensation:
  - Massive bosons in 2 dimensions
  - Massive bosons in 3 dimensions
  - Massive bosons in 4 dimensions
  - Massless bosons in 2 dimensions
  - Massless bosons in 3 dimensions
  - Massless bosons in 4 dimensions