Physics 831 Practice Quiz #5 - Monday, Dec. 3

YOUR NAME:____________________________

FUN FACTS TO KNOW AND TELL

$$\begin{aligned}
\int_0^\infty dx\ \frac{x^{n-1}}{e^x-1} &= \Gamma(n)\zeta(n), \qquad \int_0^\infty dx\ \frac{x^{n-1}}{e^x+1} = \Gamma(n)\zeta(n)\left[1-(1/2)^{n-1}\right],\\
\zeta(n) &\equiv \sum_{m=1}^{\infty} m^{-n}, \quad \Gamma(n)\equiv(n-1)!,\\
\zeta(3/2) &= 2.612375..., \quad \zeta(2)=\frac{\pi^2}{6}, \quad \zeta(3)=1.20205..., \quad \zeta(4)=\frac{\pi^4}{90},\\
&\int_{-\infty}^{\infty} dx\ e^{-x^2/2}=\sqrt{2\pi}, \qquad \int_0^\infty dx\ x^n e^{-x}=n!
\end{aligned}$$

1. Consider the virial expansion,

$$P=\rho T+\rho T\sum_{n=2,\cdots} A_n\left(\frac{\rho}{\rho_0}\right)^{n-1}.$$

For a gas of identical non-interacting bosons, A_2 is: (circle one)

greater than zero **zero** **less than zero**

2. A spherically symmetric molecule developed by super scientists at the prestigious University of Notre Dame has mass m and breathing mode excitations of $n\hbar\omega$, i.e., each mode is non-degenerate. Here, $n=0,1,2,\cdots$. A dilute gas of such molecules is kept at temperature T and chemical potential μ. Calculate the number density in terms of μ, $\beta=1/T$, m and $\hbar\omega$.

3. Suppose one has calculated a partition function,

$$Z=\text{Tr}\ e^{-\beta H}, \quad H=SCF-\mu\vec{B}\cdot\vec{S},$$

where SCF is some complicated function and $\vec{S}$ is the net spin of the system. Further assume that after performing all the fancy calculations that

$$\ln Z=N\ln[2e^z\cosh(a\beta\mu B)],$$

where a and z are functions of the temperature, and N is the number of sites.

(a) Find the average spin per site $\langle S_z\rangle$ as a function of β, μB, a, z? Assume B points along the z axis.

(b) What is the fluctuation of the spin per site $\langle S_z^2\rangle$ when $B=0$?