Group II problems, Dec. 9, 2016

DO NOT WRITE YOUR NAME OR STUDENT NUMBER ON ANY SHEET!

FUN FACTS TO KNOW AND TELL

$$\int_0^\infty dx\ \frac{x^{n-1}}{e^x-1} = \Gamma(n)\zeta(n), \qquad \int_0^\infty dx\ \frac{x^{n-1}}{e^x+1} = \Gamma(n)\zeta(n)\left[1-(1/2)^{n-1}\right],$$

$$\zeta(n) \equiv \sum_{m=1}^\infty m^{-n}, \quad \Gamma(n) \equiv (n-1)!,$$

$$\zeta(3/2) = 2.612375..., \quad \zeta(2) = \frac{\pi^2}{6}, \quad \zeta(3) = 1.20205..., \quad \zeta(4) = \frac{\pi^4}{90},$$

$$\int_{-\infty}^\infty dx\ e^{-x^2/2} = \sqrt{2\pi}, \qquad \int_0^\infty dx\ x^n e^{-x} = n!$$

Section 4.1 - 4.3

Thomas Chuna, Jenn Ranta

December 6, 2016

Problem: Write the partition function, Z, for a finite number, N, of indistinguishable diatomic particles of mass m at temperature T. Account for translation, vibration, and non-degenerate rotation. Assume that both the translation and rotation can be handled in the classical limit $\left(\text{i.e. } E_{\text{trans}} = \frac{p^2}{2m} \text{ and } E_{\text{rot}} = \frac{\hbar^2 l^2}{2I}\right)$, and assume that the temperature of the system is such that there is only a *finite* number, K, of vibrational modes. Useful identity in solving this problem: $\sum_{i=\alpha}^{\omega} r^i = \frac{r^\alpha - r^{\omega+1}}{1-r}$.

Tommy Tsang and Robert Elder

A two-dimensional system of particles expands according to ideal hydrodynamics. Initially, the two-dimensional density has a Gaussian profile

$$\rho(r, t=0) = \rho_0 e^{-r^2/2R_0^2} \,.$$

The entropy per particle in three dimensions is

$$\frac{S}{N} = \sigma = \frac{5}{2} + \ln\left(\frac{1}{\rho}\left(\frac{mT}{2\pi\hbar^2}\right)^{3/2}\right).$$

a) What is the entropy per particle in two dimensions? Give your answer in terms of *m*, *T*, and the two-dimensional density, ρ.

b) Show that for constant entropy

$$TR^2 = T_o R_0^2$$

PHY 831: Lecture Notes §4.7-4.10

Carl Fields & David Witalka

9 December 2016

Assume the probability of n bosons being in a specific phase space cell is $p_{\mathrm{n}} \propto e^{-n\beta(\epsilon-n)}$.

Find an expression for the entropy in terms of the phase space density **f**.

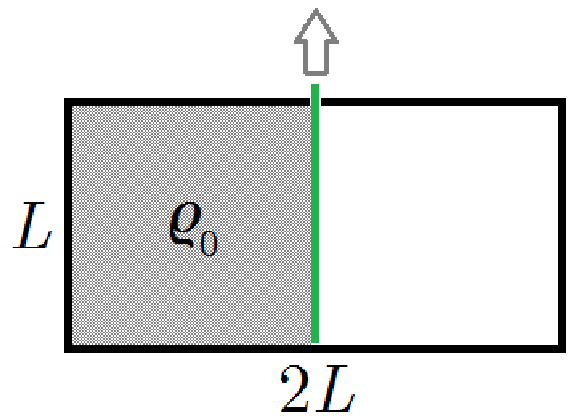
Recall $\Sigma n x^{\mathrm{n}} \equiv x\frac{\partial}{\partial x}\Sigma x^{\mathrm{n}}$.

Brent Yelle and Dustin Frisbie

Consider the system below, where equal-sized 2D boxes (each side of length L) are separated by an impermeable divider. The left side is filled with a homogeneous gas of constant density ϱ_0 and diffusion constant D, whereas the right side is a vacuum. At time $t = 0$, the divider vanishes from existence, and the gas begins to diffuse into the right cavity. Find the density in the system as a function of time.

Recall the diffusion equation:

$$\frac{\partial \rho}{\partial t} = D\nabla^2 \rho.$$



Section 5.1

Ahmed Yousif, Forrest Glines

Consider a one-dimensional array of N coupled harmonic oscillators in length L. The oscillator's movement is also confined to one dimension. Compression modes are characterized by a speed of sound c_s

(a) Solve for the Debye frequency, ω_D, in terms of ω_0, N, L, and c_s

(b) For $T \ll \omega_D$ find the specific heat,

$$C \equiv \frac{1}{L}\frac{d\langle E\rangle}{dT}.$$

(c) What is $C\,(T \to \infty)$?

(d) If the system also had electrons in addition to the harmonic oscillators, what dominates the specific heat at low temperatures, electrons or phonons, assuming

(i) harmonic oscillators can move in one dimension, as above:

(ii) assuming harmonic oscillators are free to move in two dimensions:

(iii) assuming harmonic oscillators are free to move in three dimensions:

(e) (Bonus) What if photons are present in the system instead, will they play a role?

CORRECTION: $\mu\beta \to \mu B$

Mara Grinder Heda Zhang

A) For a finite two-dimensional lattice of size L_x=2 x L_y=2, construct the 4x4 transfer matrix between neighboring columns. Hint: the transfer matrix $P_{i,i+1}$ in one dimensions is:

$$P_{i,i+1} = \begin{bmatrix} e^{\beta(J+\mu\beta)} & e^{-\beta J} \\ e^{-\beta J} & e^{\beta(J-\mu\beta)} \end{bmatrix}$$

B) Consider a one-dimensional Ising model. Label each of the following statements as true or false.

a. In the exact solution there is no phase transition. ____________

b. In the mean-field solution there is no phase transition. ____________

c. In the mean-field solution, the critical exponents are the same for the one-dimensional and two-dimensional solutions. ____________

1. A liquid-gas system coexists in equilibrium.

a. The surface free energy of such a system is given by

$$\frac{F}{A} = \sqrt{2\varkappa}\int_{\rho_{gas}}^{\rho_{liquid}} d\rho\,\sqrt{P_0 - P + (\mu - \mu_0)\rho}.$$

If the function $\Delta\boldsymbol{\psi} \equiv P_0 - P + (\mu - \mu_0)\rho$ can be approximated with the following form,

$$\Delta\psi = \frac{C}{2}[(\rho - \rho_c)^2 - \alpha^2]^2,$$

find the surface free energy.

b. What is the surface free energy in the limit T>>T_c?

PHY 831

Dana Koeppe, Xi Dong, Victor Aguilar

December 7, 2016

Problem 1 *Ising Model Stuff*

Is the Ising Model an example of discrete or continuous symmetry?

. . . .

Problem 2 *Ferromagnet*

In a two-dimensional ferromagnetic substance, spins become aligned when the temperature is lowered below the critical temperature, even though the material is in a field-free region. This is an example of: (circle all correct answers)

- spontaneous symmetry breaking
- explicit symmetry breaking
- breaking a continuous symmetry
- breaking a discrete symmetry

. . . .

PHY 831 Review Questions: Sections 6.5-6.8

Tong Li and Brenden Longfellow

1. For which of the following dimensions is Landau theory (mean field theory) guaranteed to be valid near the critical temperature according to the Ginzburg criteria? (choose the best answer)
   a) 2
   b) 3
   c) 4
   d) 5

2. Suppose there is a complex field ϕ with free energy density f of the mexican hat form:

$$f = \frac{A}{2}|\phi|^2 + \frac{B}{4}|\phi|^4 \tag{1}$$

Here, $A = at$ where a is a positive constant and $t = \frac{T-T_c}{T_c}$. Recall that $|\phi|^2 = \phi\phi^* = \phi_{real}^2 + \phi_{imaginary}^2$. There is no external field applied.

As the temperature T is lowered from above to below the critical temperature T_c, is the symmetry breaking continuous or discrete?

Is the symmetry breaking explicit or spontaneous?