EXTRA CREDIT ASSIGNMENT Due 11:59 PM Sunday, Dec. 10 2023
OPTION #3

Consider the Sun, Earth and Moon. Fix the sun at the center of the coordinate system. Assume that the moon and Earth are originally in the same $z = 0$ plane, with Earth having a position $x_0 = 152,100,000$ km, $y_0 = 0$. At this time the moon is placed in a position for a perfect solar eclipse, with the center of the moon positioned 405,800 km from Earth. The initial velocities of Earth and the Moon are:

$$V_y^{(\mathrm{Earth})} = 29.3017\ \mathrm{km/s},\ \ V_z^{(\mathrm{Earth})} = -0.00106\ \mathrm{km/s} \tag{1}$$

$$V_y^{(\mathrm{Moon})} = 28.3357\ \mathrm{km/s},\ \ V_z^{(\mathrm{Moon})} = 0.08601\ \mathrm{km/s.} \tag{2}$$

1. Write equations of motion for Earth and its moon, which include the gravitational forces of Earth with the Sun, the moon with the Sun, and Earth with the moon.

2. Solve the equations of motion for the Earth and moon numerically. To demonstrate conservation of energy as a function of time, plot the energy as a function of time.

You can download a C++ template of the program at the course webpage.

Distances:
Earth to Moon: 405696 km (apogee) and 363104 km (perigee)
Earth to Sun: 152,100,000 km (apogee) and 147,100,000 km (perigee)

Speeds:
Earth's speed about Sun: 29.29 km/s (apogee) and 30.29 km/s (perigee) Moon's speed around Earth: 0.970 km/s (apogee) and 1.082 km/s (perigee)
Masses:
Earth mass 5.972×10^{24} kg
Moon mass 7.348×10^{22} kg
Sun's mass 1.989×10^{30} kg