Driven Harmonic Oscillator

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Question

1) Suzie and Sally are playing on a swing set during their recess break after physics class. Suzie asks Sally the following questions after gaining some insight on the mechanics of a swing set in today's physics lesson.
    a. "Sally, can you draw out a free body diagram of what happens when I am swinging?"
    b. "Describe to me what happens if you start pushing me on the swing? What is the frequency if you push me in equal time intervals?" (assume the force is a sinusoidal driving force).
    c. "Draw a new diagram of what happens when you push me!"
    d. "Why do I go higher and higher even though you push the same amount?"

Conceptualizing

What basic system is being described?

A simple Pendulum at first

What happens if you add an external force?

This pendulum of a swing set becomes a driven harmonic oscillator!!

Equation of motion:

$$m\ddot{x} + b\dot{x} + kx = F(t)$$

- m is the mass of the object
- b is the damping coefficient
- k is the spring constant
- x is the position of the object
- F(t) is the external force

a)

The free body diagram can be drawn using the given equations from our exam, specifically the ones to the right and basic concepts like Fgrav = mgh

$$F(t) = \frac{f_0}{2} + \sum_n f_n \cos(n\omega t) + g_n \sin(n\omega t),$$
$$f_n = \frac{2}{\tau} \int_{-\tau/2}^{\tau/2} dt\ F(t) \cos(2n\pi t/\tau),$$
$$g_n = \frac{2}{\tau} \int_{-\tau/2}^{\tau/2} dt\ F(t) \sin(2n\pi t/\tau).$$

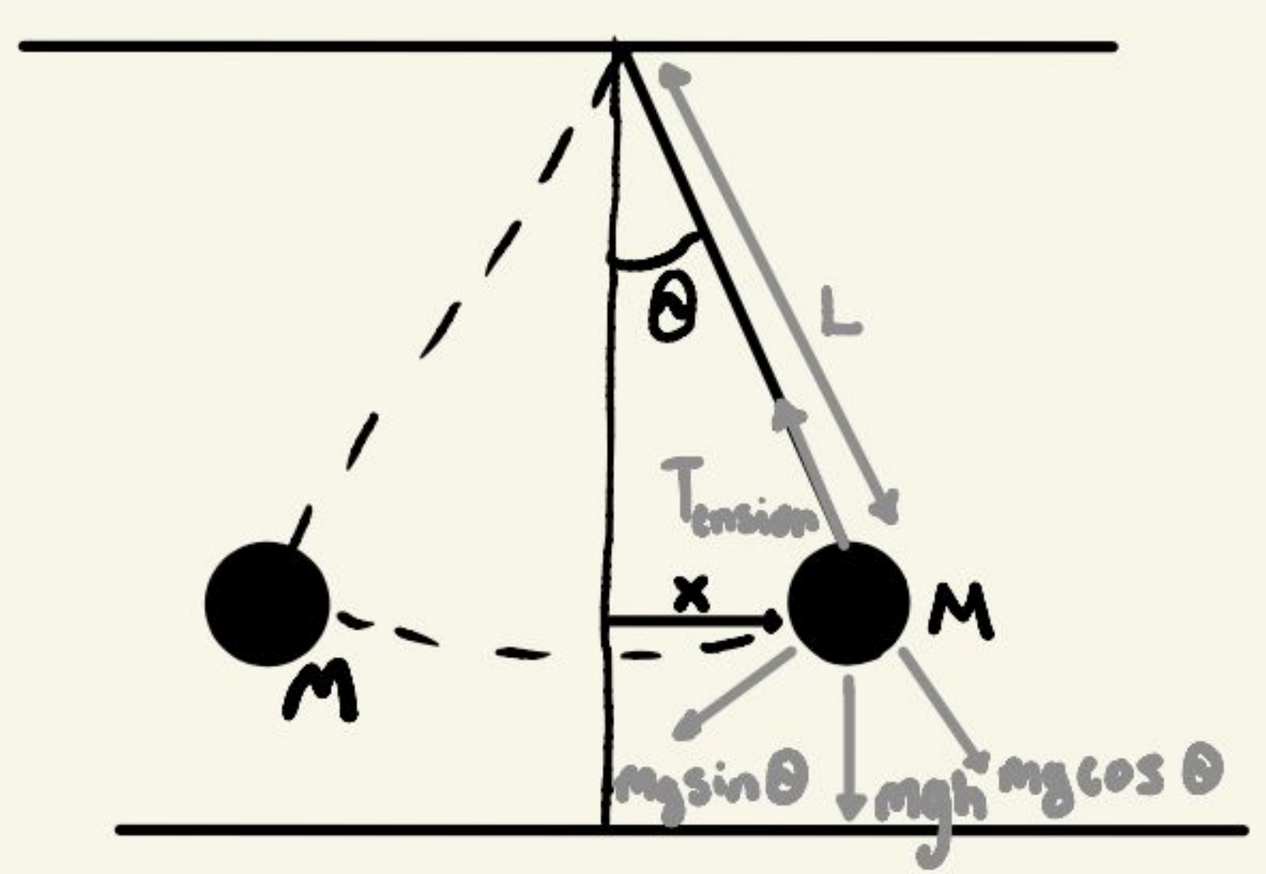
FYI: For the differential equation

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0,$$

the solutions are

$$\begin{aligned}
x &= A_1 e^{-\beta t} \cos\omega' t + A_2 e^{-\beta t} \sin\omega' t \quad \omega' = \sqrt{\omega_0^2 - \beta^2} \quad \text{(under damped)} \\
x &= A e^{-\beta t} + B t e^{-\beta t}, \quad \text{(critically damped)} \\
x &= A_1 e^{-\beta_1 t} + A_2 e^{-\beta_2 t}, \quad \beta_i = \beta \pm \sqrt{\beta^2 - \omega_0^2}, \quad \text{(over damped).}
\end{aligned}$$

a)



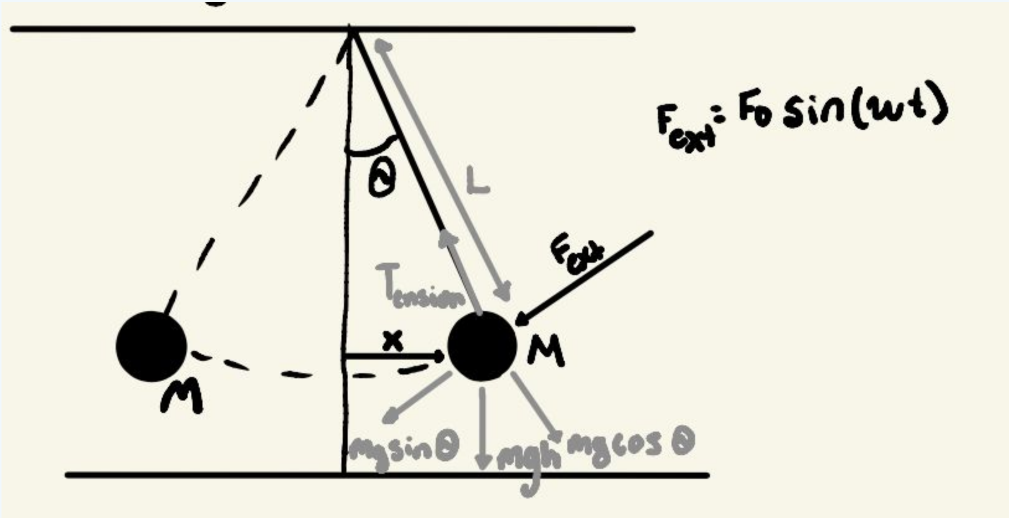
Mimics simple pendulum when Sally is swinging

b) what happens when force is applied?

When Sally begins to push Suzie's swing, Sally applies external driving force with F= Fo Sin(wt). The frequency at which sally pushes can be described as w= g/L the natural resonance frequency of the oscillator.

c) so what's different?

The only difference between this diagram and the previous is that there is a new force acting on the mass which is a sinusoidal driving force. Pushing the mass forward.



d) How does applying a force with the same frequency as the oscillator affect the motion?

Applying a force with the same resonant frequency of the swing means that the force is in phase with the motion of the “pendulum” because of this the force is opposing the effects of gravity/friction/air resistance on the swing. This Amplitude or height is given with the following equation. The amplitude grows even if the work done per period is small.

A = (Fo/m) / sqrt[(wo^2-w^2)+(Betaw/m)^2]