Problem:

Consider a system consisting of a mass m attached to a spring with a spring constant k. The system is also subjected to a driving force $F(t) = F_0 cos(\omega t)$, where F_0 is the amplitude of the driving force and ω is the driving frequency.

Question 1:

Derive the equation of motion for the system.

Solution:

Newton's second law of motion states that the force acting on an object is equal to the product of its mass and its acceleration. In this case, the force acting on the mass is the sum of the spring force and the driving force:

$$F = ma = -kx + F_0 \cos(\omega t)$$

Rearranging the equation, we get the equation of motion for the system:

$$m\ddot{x} + kx = F_0 \cos(\omega t)$$

Question 2:

Find the particular solution for the system's displacement $x(t)$.

Solution:

The particular solution for the system's displacement $x(t)$ is the solution that satisfies the inhomogeneous equation of motion. To find the particular solution, we can assume a solution of the form:

$$x(t) = A \cos(\omega t + \phi)$$

where A is the amplitude of the oscillation and ϕ is the phase difference between the driving force and the system's displacement. Substituting this assumed solution into the equation of motion, we get:

$$-mA^2\omega^2 \cos(\omega t + \phi) + kA \cos(\omega t + \phi) = F_0 \cos(\omega t)$$

Equating the coefficients of $cos(\omega t)$, we get:

$$-mA^2\omega^2 + kA = F_0$$

Solving for A, we get:

$$A = \frac{F_0}{k - m\omega^2}$$

Equating the coefficients of $sin(\omega t)$, we get:

$$-mA^2\omega^2\phi\sin(\omega t + \phi) + kA\phi\sin(\omega t + \phi) = 0$$

Since $sin(\omega t + \phi) \neq 0$ for all values of t, we must have:

$$-mA^2\omega^2\phi + kA\phi = 0$$

This equation is satisfied for any value of ϕ, so we can arbitrarily set $\phi = 0$.
Therefore, the particular solution for the system's displacement $x(t)$ is:

$$x(t) = \frac{F_0}{k - m\omega^2}\cos(\omega t)$$