A train of initial mass M_0 is moving at an initial speed of v_0 along a frictionless track. As the train travels, it picks up passengers at a constant rate of p passengers per unit distance. Each passenger has a mass of m_p.

(a) Determine the train's speed as a function of the distance traveled x.

(b) Express the train's position as a function of time.

(c) Derive an expression for the train's speed as a function of time.

General Solution:

a) Total inelastic collision, initial momentum = final momentum:

$$\rho_i = M_0 v_0 \quad \rho_f = M(x)(v(x))$$

$$\rho = (M_0 + m_p p x)v(x) = M_0 v_0$$

$$v(x) = \frac{M_0}{M_0 + m_p p x} v_0$$

b) To find Oliver's position as a function of time, we need to integrate the velocity function with respect to time:

$$\frac{dx}{dt} = \frac{M_0}{M_0 + m_p p x} v_0$$

$$M_0 v_0 t = \int_0^x dx' \, (M_0 + m_p p x') = M_0 x + m_p p \frac{x^2}{2}$$

$$x(t) = \frac{-M_0 + \sqrt{M_0^2 + 2 m_p p M_0 v_0 t}}{m_p p}$$

c) To find Oliver's position as a function of time, we need to take the derivative of the trains position as a function of time:

$$v(t) = \frac{dx}{dt} = \frac{1}{m_p p} \frac{m_p p v_0}{(M_0^2 + 2 m_p p M_0 v_0 t)^{1/2}} = \frac{M_0 v_0}{(M_0^2 + 2 m_p p M_0 v_0 t)^{1/2}}$$