your names______________________________

Physics 321 Exercise 5 - Monday, Oct. 9

FYI: From lectures notes (Eq. 4.14), the magnitude of the tidal force acting on a small mass δm on Earth due to the Moon is

$$F_{\text{tidal}} = 2\frac{GM\delta m}{D^3}z,$$

where D is distance to the moon, which has mass M, and z is the distance from Earth's center.

Consider a spherical asteroid made of rock with density $\rho_a = 2$ g/cm^3 and radius a. It is in a circular orbit of radius r about Saturn, which has mass M_S. A pebble of mass δm lies on the surface of the asteroid, either at a point facing, or opposite the direction of Saturn.

1. In terms of G, M_S, r, a and δm, express the tidal force on the pebble. The tidal force is the difference between the gravitational force from Saturn acting on the pebble vs the force it would experience if it were located at the center of the asteroid. Keep only the first order term when expanding in terms of a. Note that the tidal force is trying to lift the pebble from the surface.

2. In terms of G, a, δm and ρ_a, what is the gravitational force acting on the pebble due to the gravitational interaction with the asteroid.

3. Equate the two forces to find the radius r at which the tidal force rips apart the asteroid. Saturn's mass is $M_S = 5.68 \times 10^{26}$ kg. Note that $G = 6.67 \times 10^{-11}$ Nm^2/kg^2. Compare your answer to the radius of Saturn, $R_S = 5.82 \times 10^4$ km.

4. What would happen to your answer for (3) if the asteroid were made of ice instead of rock?

5. Express your answer for (3) in terms of R_S, ρ_a and the density of Saturn ρ_S. Note the density of Saturn is 0.9 gm/cm^3. It would float in your bathtub, if your bathtub were large enough. The average density of Jupiter is 1.3 g/cm^3 and the average density of Earth is 5.5 g/cm^3.

6. If a small moon and planet slowly spiral toward each other. If the moon and planet have the same density, will the moon be torn apart by tidal forces before reaching the surface of the planet?

7. Two planets of the same mass slowly spiral toward each other. If the two planets have the same density, will they be torn apart by tidal forces before contacting one another?