your name(s)____

Physics 321 Exercise 4 - Monday, Oct. 1

Consider a periodic force, $F(t+\tau) = F(t)$, where the form is that of a square wave,

$$F(t) = \begin{cases} -F_0, & -\tau/2 < t < 0 \\ F_0, & 0 < t < \tau/2 \end{cases}$$

$F(t)$ can be expressed in terms of its Fourier components,

$$F(t) = \frac{f_0}{2} + \sum_{n>0} f_n \cos(n\omega t) + g_n \sin(n\omega t), \quad \omega \equiv 2\pi/\tau.$$

Use the expressions

$$f_n = \frac{2}{\tau}\int_{-\tau/2}^{\tau/2} dt\ F(t)\cos(n\omega t),$$
$$g_n = \frac{2}{\tau}\int_{-\tau/2}^{\tau/2} dt\ F(t)\sin(n\omega t).$$

1. (5 pts) Which coefficients are zero?
2. (5 pts) Find the non-zero coefficients, f_n and g_n.
3. (5 pts) Plot your result for the sum, $F(t)/F_0$, as a function of t/τ. Make 3 plots, with the cutoff on the sum set at $n_{\max} = 5, 50, 500$. Plot your result for $-2 < t/\tau < 2$.
4. (5 pts) Now consider the force acting on a mass m in a harmonic oscillator, with fundamental frequency f_0 and damping rate β. The particular solution is:

$$\begin{aligned} x_p(t) &= \frac{f_0}{2k} + \sum_{n>0} \alpha_n \cos(n\omega t - \delta_n) + \beta_n \sin(n\omega t - \delta_n), \qquad (1) \\ \alpha_n &= \frac{f_n/m}{\sqrt{((n\omega)^2 - \omega_0^2)^2 + 4\beta^2 n^2\omega^2}}, \\ \beta_n &= \frac{g_n/m}{\sqrt{((n\omega)^2 - \omega_0^2)^2 + 4\beta^2 n^2\omega^2}}, \\ \delta_n &= \tan^{-1}\left(\frac{2\beta n\omega}{\omega_0^2 - n^2\omega^2}\right). \end{aligned}$$

For $F_0 = 50$ N, $\tau = 0.5$ s, $\beta = 1.5$ Hz, $f_0 = 0.75$ Hz and $m = 100$ g, plot $x_p(t)$ for $-20 < t < 20$ s.

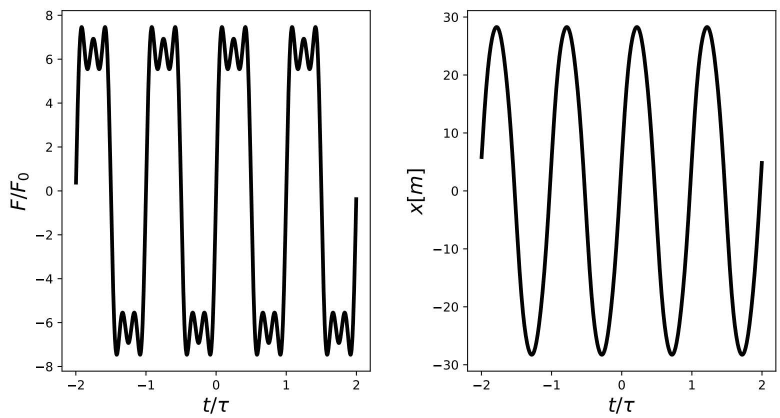
Solutions:

1. All f_n are zero, and $g_n = 0$ for even n
2. Doing the integral

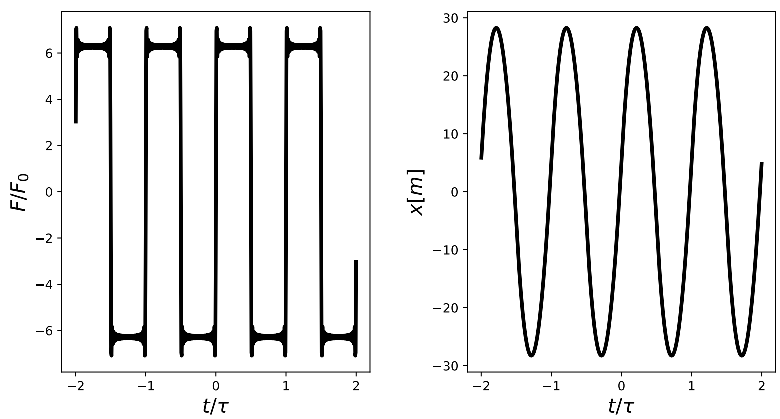
$$g_{n=\mathrm{odd}} = \frac{4F_0}{n\tau}$$

Plots for (3) and (4)

For $n_{\mathrm{max}} = 5$,



For $n_{\mathrm{max}} = 50$,



For $n_{\mathrm{max}} = 500$,