

your name(s) \_\_\_\_\_

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*Physics 321 Exercise 4 - Monday, Oct. 1*

Consider a periodic force,  $F(t + \tau) = F(t)$ , where the form is that of a square wave,

$$F(t) = \begin{cases} -F_0, & -\tau/2 < t < 0 \\ F_0, & 0 < t < \tau/2 \end{cases}$$

$F(t)$  can be expressed in terms of its Fourier components,

$$F(t) = \frac{f_0}{2} + \sum_{n>0} f_n \cos(n\omega t) + g_n \sin(n\omega t), \quad \omega \equiv 2\pi/\tau.$$

Use the expressions

$$f_n = \frac{2}{\tau} \int_{-\tau/2}^{\tau/2} dt F(t) \cos(n\omega t),$$

$$g_n = \frac{2}{\tau} \int_{-\tau/2}^{\tau/2} dt F(t) \sin(n\omega t).$$

1. (5 pts) Which coefficients are zero?
2. (5 pts) Find the non-zero coefficients,  $f_n$  and  $g_n$ .
3. (5 pts) Plot your result for the sum,  $F(t)/F_0$ , as a function of  $t/\tau$ . Make 3 plots, with the cutoff on the sum set at  $n_{\max} = 5, 50, 500$ . Plot your result for  $-2 < t/\tau < 2$ .
4. (5 pts) Now consider the force acting on a mass  $m$  in a harmonic oscillator, with fundamental frequency  $f_0$  and damping rate  $\beta$ . The particular solution is:

$$x_p(t) = \frac{f_0}{2k} + \sum_{n>0} \alpha_n \cos(n\omega t - \delta_n) + \beta_n \sin(n\omega t - \delta_n), \quad (1)$$

$$\alpha_n = \frac{f_n/m}{\sqrt{((n\omega)^2 - \omega_0^2)^2 + 4\beta^2 n^2 \omega^2}},$$

$$\beta_n = \frac{g_n/m}{\sqrt{((n\omega)^2 - \omega_0^2)^2 + 4\beta^2 n^2 \omega^2}},$$

$$\delta_n = \tan^{-1} \left( \frac{2\beta n \omega}{\omega_0^2 - n^2 \omega^2} \right).$$

For  $F_0 = 50$  N,  $\tau = 0.5$  s,  $\beta = 1.5$  Hz,  $f_0 = 0.75$  Hz and  $m = 100$  g, plot  $x_p(t)$  for  $-20 < t < 20$  s.