your name(s)______________________________

*Physics 321 Exercise 2 - Monday, Sep. 18*

A particle of mass $m$ is in a damped harmonic oscillator. The spring constant is $m\omega_0^2$, and the damping force is $-bv$ with $\beta = b/2m$. Deriving the equations of motion,

$$\begin{aligned} m\ddot{x} &= -m\omega_0^2 x - b\dot{x}, \\ \ddot{x} &= -\omega_0^2 x - 2\beta\dot{x}. \end{aligned}$$

Assume time is discretized such that $t_n = n\Delta t$, and the $x_n$, $v_n$ and $a_n$ refer to the position, velocity and acceleration at time $t_n$. Beginning with

$$\begin{aligned} v_n &= \frac{x_{n+1} - x_{n-1}}{2\Delta t}, \\ a_n &= \frac{x_{n+1} - 2x_n + x_{n-1}}{(\Delta t)^2}, \end{aligned}$$

and

$$a_n = F(x_n)/m,$$

1. (5 pts) Show that this becomes

   $$x_{n+1} = \frac{\left[2 - \omega_0^2(\Delta t)^2\right] x_n - [1 - \beta\Delta t]\, x_{n-1}}{1 + \beta\Delta t}$$

   This can then be solved iteratively for all $n \geq 2$. But, to get started, one needs to know both both $x_0$ and $x_1$. If the initial velocity, $v_0$ is given, one can set $x_1 = x_0 + v_0\Delta t + a_0\Delta t^2/2$.

2. (5 pts) A particle of mass, $m = 100$ grams, moves in a harmonic oscillator where the fundamental frequency (no damping) is $f_0 = 0.5$ Hz. The damping rate is $\beta = b/2m = 0.5$ Hz. The particle is initially at the origin with initial velocity $v_0 = 5.0$ m/s.

   Write a program that numerically solves for $x(t)$ for $0 \leq t \leq 10$ s. To get started, one needs $x_1$, which by the formula above becomes.

   $$x_1 = v_0 t - \frac{1}{2}\beta v_0 (\Delta t)^2.$$

   Plot $x(t)$.

3. (5 pts) Repeat for $\beta = 1.0, 2.0$ and $4.0$ Hz. (Note: the oscillator becomes critically damped when $\beta > \omega_0$)