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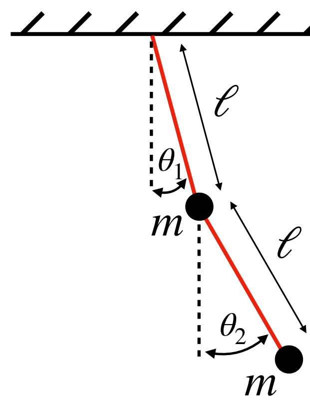
Exercise, Monday and Wednesday, Nov. 20,22, 2023



Consider the double pendulum pictured above, where both rods have negligible mass. Let x_1, y_1 and x_2, y_2 refer to the positions of the upper and lower mass respectively.

1. (5 pts) Write $\dot{x}_1$ and $\dot{y}_1$ in terms of θ_1 and $\dot{\theta}_1$.
2. (5 pts) Write $\dot{x}_2$ and $\dot{y}_2$ in terms of θ_1, θ_2, $\dot{\theta}_1$ and $\dot{\theta}_2$.
3. (5 pts) Write the total kinetic energy in terms of angles and their derivatives.
4. (5 pts) Write the potential energy in terms of the angles.
5. (10 pts) Write the equations of motion.
6. (5 pts) Rewrite the equations of motion in the small-angle limit, using $\omega_0 \equiv \sqrt{g/\ell}$.
7. (5 pts) Assume the solutions to the equations above are of the form

$$\theta_1 = Ae^{i\omega t},$$
$$\theta_2 = Be^{i\omega t}.$$

   Use the equations of motion (in the small-angle limit) to write corresponding equations in terms of ω.

8. (5 pts) Solve for ω. There should be two solutions to the quadratic equation, ω_+ and ω_-.
9. (5 pts) For each value of ω, solve for A and B, i.e. find A_+, B_+ and A_-, B_-.

– Last two parts represent material not on final exam –

10. (5 pts) Express the equations of motion in (7) in matrix form,

$$M\omega^2 \begin{pmatrix} a \\ b \end{pmatrix} = K \begin{pmatrix} a \\ b \end{pmatrix}.$$

I.e., what are the matrices M and K?

11. (5 pts) Using the provided python template (one of the templates from the course page) as a starting point, find the two eigenvalues, λ_1 and λ_2, such that

$$\left(\frac{1}{\omega_0^2} M^{-1} K\right) \begin{pmatrix} a_i \\ b_i \end{pmatrix} = \lambda_i \begin{pmatrix} a_i \\ b_i \end{pmatrix}.$$

Here, the eigenvalues, λ_i, are related to the frequencies by $\omega_i^2 = \lambda_i \omega_0^2$. The eigenvectors are

$$\begin{pmatrix} a_i \\ b_i \end{pmatrix}.$$

The eigenvectors calculated in your python script are normalized such that $a_i^2 + b_i^2 = 1$. Show that the eigenvalues and eigenvectors correspond to your answers in (8) and (9).

Solutions can be found as example in Chapter 6.